

Full Counting Statistics of Electron Tunneling through Coherently Coupled Quantum Dots: Exchange Interaction Effect on Shot Noise

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We investigate the zero-frequency shot noise of electronic tunneling through a single quantum dot (SQD) and coherently coupled quantum dots (CQD) taking into account the Coulomb interaction. Within Hartree-Fock approximation, the analytical expressions of current and zero-frequency shot noise are self-consistently derived in the framework of full counting statistics for the both systems. We demonstrate that the correction term of zero-frequency shot noise induced by the intradot Coulomb interaction is almost negligible compared to the noninteracting shot noise in a SQD, while in a CQD the interplay of the interdot coherence fluctuations and strong interdot Coulomb interaction can induce a super-Poissonian noise even in the symmetric case.

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I. INTRODUCTION

The dynamics of electron transport through the quantum dot (QD) system is one of the most interesting topics in mesoscopic physics, which has motivated many experimental investigations, such as ones on the structures of barriers with localized states,^{1,2} the single quantum dot (SQD)³⁻⁷ and the double or multiple quantum dots,⁸⁻¹³ and also a lot of theoretical research.¹⁴⁻²⁵ In particular, the shot noise, resulting from charge discreteness and stochastic transfer process, serves as an important indicator for the dynamic property in the mesoscopic structures,¹⁴ as well as a revelation of a lot of other information, such as the charge unit^{26,27} and entanglement^{28,29} of the transferred charges. The enhancement of shot noise is an important signature for the non-Poisson process. The classical Poisson process requires shot noise S follows the expression $S = 2eI$, where e refers to the unit charge and I the current, while in quantum transport the charge distribution is proved to follow the binomial statistics.³⁰ It has been reported that the electronic tunneling through a single quantum dot with multiple energy levels and a coupled quantum dots (CQD) system turned out to be super-Poisson ($S > 2eI$).^{6-8,12} For the system of CQD, the interdot coherence and Coulomb interaction are essential factors to determine the shot noise characteristics of this system, thereby arousing great research interests^{8,12,21-24} and deserving more research investigations.

Over the past decades, many theoretical attempts have been devoted to understanding of the super-Poisson process in the QD system, especially the effects of Coulomb blockade in coherently coupled quantum dots system on the enhancement of zero-frequency shot noise. It is worthwhile to point out that when only the interdot coherent coupling^{31,32} or the Coulomb blockade³³⁻³⁵ effect is taken into account, the noise-to-current ratio (Fano factor) is found to be suppressed. However, the joint ef-

fect of the coherence and the Coulomb interaction has been reported to enhance the shot noise. Aghassi *et al.*²² demonstrated the appearance of super-Poissonian noise for symmetric configuration of a CQD taking account of both intra- and inter-dot Coulomb repulsions if the vertical energy gap is relatively smaller than the sequential energy gap and the applied bias voltage is below the sequential threshold. Nevertheless for the strongly broken symmetric case, they reported the observation of both super-Poissonian noise and negative differential conductance in the region with the bias above the sequential tunneling threshold. Kießlich *et al.*¹² attributed the noise enhancement of CQD to the interplay of strong Coulomb repulsion and quantum coherence, and showed the high sensitivity of such mechanism to the decoherence caused by electron-phonon scattering. One of the authors, Dong,^{23,24} predicted a giant Fano factor of electronic tunneling through a parallel CQD in the strong Coulomb blockade regime due to the interference between two transport path. However, all the works mentioned above employed the technique of master equation, whether modified or not, whose validity is not ensured in the small bias region and strong interacting system. Out of the application boundary of master equation, more appropriate techniques are expected. For instance, López *et al.*²⁵ adopted nonequilibrium Green functions (NGF) with slave-boson mean-field theory to describe correlated double quantum dots system.

In this work, within the framework of NGF and Hartree-Fock (HF) decoupling approximation, we adopt the full counting statistics (FCS) theory³⁶⁻³⁹ to investigate the zero-frequency shot noise of electronic tunneling through a series CQD with symmetric configuration in strong Coulomb blockade regime. FCS was introduced to describe the probability distribution of charge transport through mesoscopic conductors and thus contained important dynamic information.³⁰ With these instruments, we study the joint effect of coherent exchange

and Coulomb interactions on the shot noise in detail. Due to this joint effect, super-Poissonian characteristic structures in shot noise, i.e. the peak and plateau, are found at the small and large bias region, respectively.

The outline of this paper is as follows. In Sec.II, we give a simplified approach to zero-frequency shot noise in single quantum dot in comparison with the work of Hershfield,⁴⁰ who performed the derivations of shot noise expression within the diagrammatic technique. In this way, we confirm the validity of our theory to some extent. In Sec.III, this technique is generalized to study the CQD system, and numerical simulations and discussions are presented. Then the summary is given in Sec.IV.

II. SINGLE QUANTUM DOT

A. Model and Approximation

We adopt the impurity Anderson model to describe the single quantum dot system including two electrodes (source and drain reservoirs) and a quantum dot, specifically as

$$H = H_L + H_R + H_D + H_T, \quad (1)$$

with

$$H_\eta = \sum_{\eta k \sigma} \epsilon_{\eta k \sigma} c_{\eta k \sigma}^\dagger c_{\eta k \sigma}, \quad (2)$$

$$H_D = \sum_{\sigma} E_0 d_{\sigma}^\dagger d_{\sigma} + U d_{\uparrow}^\dagger d_{\uparrow} d_{\downarrow}^\dagger d_{\downarrow}, \quad (3)$$

$$H_T = \sum_{\eta k \sigma} \left[V_{\eta k} e^{i\lambda_\eta(t)/2} d_{\sigma}^\dagger c_{\eta k \sigma} + H.c. \right]. \quad (4)$$

Here, H_η ($\eta = \{L, R\}$) models the left and right leads with the noninteracting approximation, where $\epsilon_{\eta k \sigma}$ denotes the single-electron energy with the momentum label (k) and spin label ($\sigma = \{\uparrow, \downarrow\}$) and $c_{\eta k \sigma}^\dagger$ ($c_{\eta k \sigma}$) creates (destroys) a corresponding electron in the electrode η . H_D describes the quantum dot with the electron energy E_0 and the Coulomb interaction parameter U , where d_{σ}^\dagger and d_{σ} are the creation and annihilation operators for the spin- σ electron on the dot. The last term H_T models the electron tunneling between leads and the dot with the coupling parameters V_{Lk} and V_{Rk} , which link the dot to the left electrode and the right one respectively. In the framework of FCS, the counting field with respect to the lead η is introduced and labeled by $\lambda_\eta(t)$. Note that in this work, the symbol $-$ denotes the forward path on Keldysh contour and the symbol $+$ denotes the backward path. Thus, on the forward or backward path $\lambda_\eta(t)$ is expressed as $\lambda_{\eta\mp}(t) = \lambda_{\eta\mp}\theta(t)\theta(\mathcal{T} - t)$ and furthermore results in $\lambda_{\eta-} = -\lambda_{\eta+} = \lambda_\eta$ in calculations of FCS. Here, \mathcal{T} denotes the measuring time during which

the counting fields are non-zero. Because of the configuration symmetry, we might as well set the counting fields between the right lead and the dot $\lambda_{R-} = \lambda_{R+} = 0$ for the convenience.³⁷⁻³⁹

For the purpose of investigating FCS, we employ the Hartree approximation to deal with the Coulomb interaction within the quantum dot, i.e. write the second term of H_D as $U\rho_{\uparrow}d_{\downarrow}^\dagger d_{\downarrow} + U\rho_{\downarrow}d_{\uparrow}^\dagger d_{\uparrow}$, where we define $\rho_{\uparrow} = \langle d_{\uparrow}^\dagger d_{\uparrow} \rangle_\lambda$ and $\rho_{\downarrow} = \langle d_{\downarrow}^\dagger d_{\downarrow} \rangle_\lambda$. It is deserved to emphasize that $\rho_{\sigma}^- \neq \rho_{\sigma}^+$, whose superscripts $-$ and $+$ indicate the position of time argument on the Keldysh contour, resulting from the fact $\lambda_{\eta-} \neq \lambda_{\eta+}$. Therefore, in this approximation the total H_D is written as

$$\begin{aligned} H_D &= \sum_{\sigma} E_0 d_{\sigma}^\dagger d_{\sigma} + U\rho_{\uparrow}d_{\downarrow}^\dagger d_{\downarrow} + U\rho_{\downarrow}d_{\uparrow}^\dagger d_{\uparrow} \\ &= \sum_{\sigma} \epsilon_0 d_{\sigma}^\dagger d_{\sigma}, \end{aligned} \quad (5)$$

where we introduce the λ -dependent noninteracting energy level ϵ_0 , which is defined as $\epsilon_0 = E_0 + U\rho$. Here, the λ -dependent electron occupation number holds the relation that $\rho = \rho_{\uparrow} = \rho_{\downarrow}$ in this non-magnetic system. However, note that $\epsilon_0^- \neq \epsilon_0^+$, from the obvious relation $\rho^- \neq \rho^+$. At the end, we point out that through this paper we use the natural units ($\hbar = e = k_B = 1$).

B. Theoretical Formulation

In this subsection, we shall first carry out the calculation to obtain the generating function $\chi(\lambda)$ or the cumulant generating function $\ln \chi(\lambda)$. Then according to the relation between cumulants and the cumulant generating function, the current and the shot noise expressions are derived respectively, actually ending up in terms of some response and correlation functions which will be defined later.

To obtain the so-called cumulant generating function $\ln \chi(\lambda)$, which takes the expression as $\chi(\lambda) = \langle T_C e^{-i \int_C H_T(t) dt} \rangle$, where T_C is the Keldysh contour ordering operator, the adiabatic method is a good choice: $\ln \chi(\lambda) = -i\mathcal{TU}(\lambda_-, \lambda_+) = -i\mathcal{TU}(\lambda, -\lambda)$, where $\mathcal{U}(\lambda_-, \lambda_+)$ is the adiabatic potential. A quite straightforward way is to take the functions $\lambda_{\pm}(t)$ to different constants as λ_{\pm} first, then substitute the expression of T_λ into the non-equilibrium Feynman-Hellmann equation, which is $\frac{\partial \mathcal{U}(\lambda_-, \lambda_+)}{\partial \lambda_-} = \left\langle \frac{\partial H_T(t)}{\partial \lambda_-} \right\rangle_\lambda$, and naturally arrive at

$$\frac{\partial \mathcal{U}(\lambda_-, \lambda_+)}{\partial \lambda_{L-}} = \frac{i}{2} \sum_{k\sigma} \langle V_{Lk} e^{i\lambda_{L-}/2} d_{\sigma}^\dagger c_{Lk\sigma} - H.c. \rangle_\lambda. \quad (6)$$

For practical reasons, we introduce the contour-order mixed GFs

$$G_{\eta k \sigma, d \sigma}(t, t') = -i \langle T_C c_{\eta k \sigma}(t) d_{\sigma}^\dagger(t') \rangle_\lambda, \quad (7)$$

$$G_{d\sigma,\eta k\sigma}(t, t') = -i\langle T_C d_\sigma(t) c_{\eta k\sigma}^\dagger(t') \rangle_\lambda. \quad (8) \quad \text{Thus, we can rewrite Eq.(6) as}$$

$$\frac{\partial \mathcal{U}(\lambda_-, \lambda_+)}{\partial \lambda_-} = \frac{1}{2} \sum_{k\sigma} \left[V_{Lk} e^{i\lambda_{L-}/2} G_{Lk\sigma, d\sigma}^{--}(t, t^+) - V_{Lk} e^{-i\lambda_{L-}/2} G_{d\sigma, Lk\sigma}^{--}(t, t^+) \right]. \quad (9)$$

where t^+ is defined as $t^+ = t + 0^+$. Note that

$$G_{\eta k\sigma, d\sigma}^{--}(t, t') = V_{Lk} \int dt_1 \left[g_{\eta k\sigma}^{--}(t, t_1) e^{-i\lambda_{\eta-}/2} G_{d\sigma}^{--}(t_1, t') - g_{\eta k\sigma}^{+-}(t, t_1) e^{-i\lambda_{\eta+}/2} G_{d\sigma}^{+-}(t_1, t') \right], \quad (10)$$

$$G_{d\sigma, \eta k\sigma}^{--}(t, t') = V_{Lk} \int dt_1 \left[G_{d\sigma}^{--}(t, t_1) e^{i\lambda_{\eta-}/2} g_{\eta k\sigma}^{--}(t_1, t') - G_{d\sigma}^{+-}(t, t_1) e^{i\lambda_{\eta+}/2} g_{\eta k\sigma}^{+-}(t_1, t') \right], \quad (11)$$

where we define the QD Green function $G_{d\sigma}(t, t') = -i\langle T_C d_\sigma(t) d_\sigma^\dagger(t') \rangle_\lambda$ and the bare lead GFs are $g_{\eta k\sigma}(t, t') = -i\langle T_C c_{\eta k\sigma}(t) c_{\eta k\sigma}^\dagger(t') \rangle_\lambda$. Substituting the above two Dyson equations into Eq.(9) and performing Fourier transformation, we obtain

$$\frac{\partial \mathcal{U}(\lambda_-, \lambda_+)}{\partial \lambda_-} = \frac{i}{2} \sum_{\sigma} \int \frac{dE}{2\pi} \left[G_{d\sigma}^{+-}(E) \Sigma_{L\sigma}^{+-}(E) - \Sigma_{L\sigma}^{+-}(E) G_{d\sigma}^{+-}(E) \right], \quad (12)$$

where with superscripts μ, ν marking the time position on the Keldysh contour ($\mu, \nu = \{-, +\}$), we define the self-energy

$$\Sigma_{\eta\sigma}^{\mu\nu}(E) = \sum_k V_{\eta k}^2 e^{i(\lambda_{\eta\mu} - \lambda_{\eta\nu})/2} g_{\eta k\sigma}^{\mu\nu}(E). \quad (13)$$

From now on, we will omit the energy argument E to save the space. Obviously, in order to obtain the adiabatic potential, we have to evaluate the GFs for the QD. Using the equation of motion (EOM) technique (details please refer to the Appendix A), we obtain

$$\hat{G}_{d\sigma} = \begin{pmatrix} G_{d\sigma}^{--} & G_{d\sigma}^{+-} \\ G_{d\sigma}^{+-} & G_{d\sigma}^{++} \end{pmatrix} = \frac{1}{\mathcal{D}} \begin{pmatrix} E - E_0 - U\rho^+ + \sum_{\eta} i\Gamma_{\eta}(f_{\eta} - \frac{1}{2}) & ie^{i\bar{\lambda}_L/2}\Gamma_L f_L + i\Gamma_R f_R \\ -i\Gamma_L e^{-i\bar{\lambda}_L/2}(1 - f_L) - i\Gamma_R(1 - f_R) & -(E - E_0 - U\rho^-) + \sum_{\eta} i\Gamma_{\eta}(f_{\eta} - \frac{1}{2}) \end{pmatrix}, \quad (14)$$

where $\bar{\lambda}_{\eta} = \lambda_{\eta-} - \lambda_{\eta+}$. Here \mathcal{D} is given as

$$\mathcal{D} = i \left[\Gamma_R(f_R - \frac{1}{2}) + \Gamma_L(f_L - \frac{1}{2}) \right] (\epsilon_0^+ - \epsilon_0^-) + (E - \epsilon_0^-)(E - \epsilon_0^+) + \Gamma^2 - \Gamma_L \Gamma_R \left[f_R(1 - f_L)(1 - e^{-i\bar{\lambda}_L/2}) + f_L(1 - f_R)(1 - e^{i\bar{\lambda}_L/2}) \right], \quad (15)$$

where f_{η} denotes the Fermi distribution function with respect to the lead η and Γ is defined as $\Gamma = (\Gamma_L + \Gamma_R)/2$ with the definition of level-width function $\Gamma_{\eta} = 2\pi \sum_k \rho(E_k) V_{\eta k}^2$ and $\rho(E_k)$ indicating the density of states. Γ_{η} can be taken as energy-independent and hence a constant with the assumption of the wide band limit.

Solve the Dyson equation and then obtain the demanded expressions of $G_{d\sigma}^{--}$ and $G_{d\sigma}^{++}$ for the evaluation of Eq.(12). To complete the calculations for the adiabatic potential, it should be noted that the Hartree approximation introduces two unknown parameters ρ^- and ρ^+ involved in the expression of \mathcal{D} and hence in that of $\hat{G}_{d\sigma}$. To solve the problem, we have to find the self-consistent equations and the corresponding solution of GFs, which we need to derive the adiabatic potential formula. According to the definition of $G_{d\sigma}^{--}(t, t')$ and $G_{d\sigma}^{++}(t, t')$, we

obtain the following self-consistent equations:

$$\rho^- = \int \frac{dE}{2\pi i} G_{d\sigma}^{--} e^{iE0^+}, \quad (16)$$

$$\rho^+ = \int \frac{dE}{2\pi i} G_{d\sigma}^{++} e^{-iE0^+}. \quad (17)$$

It is found that the to-be-solved parameters ρ^- and ρ^+ also contained in the expressions for $G_{d\sigma}^{--}$ and $G_{d\sigma}^{++}$.

Then one can do the expansion of ρ^- and ρ^+ in the Eq.(16) and Eq.(17) with respect to the counting field λ to some proper orders according to the aim of the calculations. In the following, we just need to expand them to the first order with respect to $(\frac{i}{2}\lambda)$:⁴¹

$$\rho^- = \rho^{(0)} + \rho^{-(1)} \left(\frac{i}{2}\lambda \right) + o(i\lambda), \quad (18)$$

$$\rho^+ = \rho^{(0)} + \rho^{+(1)} \left(\frac{i}{2}\lambda \right) + o(i\lambda). \quad (19)$$

Unrelated to the external measuring field, the zero-order term is the result from the mean-field approach. By contrast, the concerned fluctuation of the mean electron occupation number induced by the Coulomb interaction is mainly described by the first-order term, and hence connected with the counting field. Actually, the two zero-order self-consistent equations hold the same form:

$$\rho^{(0)} = \int \frac{dE}{2\pi} \frac{\Gamma_L f_L + \Gamma_R f_R}{\mathcal{D}^{(0)}}, \quad (20)$$

where $\mathcal{D}^{(0)} = (E - \epsilon_0^{(0)})^2 + \Gamma^2$ and $\epsilon_0^{(0)} = E_0 + U\rho^{(0)}$. Obviously, $\rho^{(0)}$ is totally determined by the Eq.(20), and will be assumed to be known in the solutions of $\rho^{-(1)}$ and $\rho^{+(1)}$. Then, let us turn to the first-order self-consistent equations, and for the purpose of symmetric forms, they are constructed as:

$$\rho^{-(1)} + \rho^{+(1)} = (\mathcal{M}_1 + \mathcal{M}_2)(\rho^{-(1)} - \rho^{+(1)}) + \mathcal{M}_3(\rho^{-(1)} + \rho^{+(1)}) + \mathcal{M}_4, \quad (21)$$

$$\rho^{-(1)} - \rho^{+(1)} = (-\mathcal{M}_1 + \mathcal{M}_5)(\rho^{-(1)} + \rho^{+(1)}) + \mathcal{M}_3(\rho^{-(1)} - \rho^{+(1)}) + \mathcal{M}_6, \quad (22)$$

where the integrals in the equations are

$$\mathcal{M}_1 = \int \frac{dE}{2\pi i} \frac{U}{\mathcal{D}^{(0)}}, \quad (23a)$$

$$\mathcal{M}_2 = \int \frac{dE}{2\pi i} \frac{2U\mathcal{A}^2}{\mathcal{D}^{(0)^2}}, \quad (23b)$$

$$\mathcal{M}_3 = \int \frac{dE}{2\pi} \frac{2U\mathcal{A}(E - \epsilon_0)}{\mathcal{D}^{(0)^2}}, \quad (23c)$$

$$\mathcal{M}_4 = \int \frac{dE}{2\pi} \frac{2\mathcal{A}\mathcal{B}}{\mathcal{D}^{(0)^2}}, \quad (23d)$$

$$\mathcal{M}_5 = \int \frac{dE}{2\pi i} \frac{2U(E - \epsilon_0)^2}{\mathcal{D}^{(0)^2}}, \quad (23e)$$

$$\mathcal{M}_6 = \int \frac{dE}{2\pi i} \frac{2(E - \epsilon_0)\mathcal{B}}{\mathcal{D}^{(0)^2}}, \quad (23f)$$

with

$$\mathcal{A} = \Gamma_L(f_L - \frac{1}{2}) + \Gamma_R(f_R - \frac{1}{2}), \quad (24a)$$

$$\mathcal{B} = \Gamma_L \Gamma_R (f_R - f_L). \quad (24b)$$

Solving the self-consistent Eq.(21) and Eq.(22), we find the solutions for $\rho^{-(1)}$ and $\rho^{+(1)}$, which account for the fluctuation of electron occupation number and contribute to the correction of the shot noise.

Until now, we have obtained the zero and first order solutions of GFs in theory if substituting the numerical calculations of $\rho^{(0)}$, $\rho^{-(1)}$ and $\rho^{+(1)}$ into $\hat{G}_{d\sigma}$. Then, it is natural to derive the formulae of the current I and the zero-frequency shot noise S . According to Eq.(16), Eq.(17) and Eqs.(A6), we arrive at the explicit expression of Eq.(12) ($\bar{\lambda}_\eta = \lambda_{\eta-} - \lambda_{\eta+}$):

$$\frac{\partial \mathcal{U}(\lambda_-, \lambda_+)}{\partial \lambda_{L-}} = -\Gamma_L \Gamma_R \int \frac{dE}{2\pi} \frac{1}{\mathcal{D}} \left[e^{i\bar{\lambda}_L/2} f_L (1 - f_R) - e^{-i\bar{\lambda}_L/2} f_R (1 - f_L) \right]. \quad (25)$$

Thus, the current can be calculated as

$$I = \frac{1}{\mathcal{T}} \frac{\partial \ln \chi(\lambda)}{\partial (i\lambda_L/2)} \Big|_{\lambda=0} = 2 \int \frac{dE}{2\pi} T(E) (f_L - f_R), \quad (26)$$

with the transmission coefficient $T(E)$ defined as $(\Gamma = (\Gamma_L + \Gamma_R)/2)$:

$$T(E) = \frac{\Gamma_L \Gamma_R}{(E - E_0 - U\rho^{(0)})^2 + \Gamma^2}. \quad (27)$$

And the zero-frequency shot noise can be evaluated as

$$S = 2 \frac{1}{\mathcal{T}} \frac{\partial^2 \ln \chi(\lambda)}{\partial (i\lambda_L/2)^2} \Big|_{\lambda=0} = S_0 + S_c. \quad (28)$$

Here, we divide the zero-frequency shot noise S into two separated parts, i.e. the mean-field current fluctuation S_0 and the correction term S_c , which is due to the intradot Coulomb interaction. Their explicit expressions are as follows:

$$S_0 = 4 \int \frac{dE}{2\pi} \{ T(E) [f_L (1 - f_R) + f_R (1 - f_L)] - T^2(E) (f_L - f_R)^2 \}, \quad (29)$$

and

$$S_c = \frac{4}{\Gamma_L \Gamma_R} \int \frac{dE}{2\pi} T^2(E) (f_L - f_R) U \left\{ (E - \epsilon_0) (\rho^{-(1)} + \rho^{+(1)}) + i[\Gamma_L (f_L - \frac{1}{2}) + \Gamma_R (f_R - \frac{1}{2})] (\rho^{-(1)} - \rho^{+(1)}) \right\}. \quad (30)$$

About twenty years ago, S. Hershfield⁴⁰ has adopted the Feynman diagram expansion technique to obtain the concerned expression of S_c by some response and correlation functions. In order to compare our derivation of S_c with this result, we begin with the definitions of the current-density response function χ_{jn} , the density-density response function χ_{nn} , the density-density correlation function S_{nn} and the current-density correlation function S_{jn} :

$$\chi_{jn} = \int dt \{ -i\theta(t) \langle [j(t), \rho(0)] \rangle \}, \quad (31a)$$

$$\chi_{nn} = \int dt \{ -i\theta(t) \langle [\rho(t), \rho(0)] \rangle \}, \quad (31b)$$

$$S_{nn} = \int dt \langle [\rho(t)\rho(0)] - \langle \rho \rangle^2 \rangle, \quad (31c)$$

$$S_{jn} = \int dt \langle [j(t)\rho(0)] + \langle \rho(t)j(0) \rangle - 2\langle j \rangle \langle \rho \rangle \rangle, \quad (31d)$$

where $j(t)$ indicates the current operator. One can define the effective distribution function of the quantum dot $f_{eff} = (\Gamma_L f_L + \Gamma_R f_R) / (\Gamma_L + \Gamma_R)$ and carry out the calculations as:

$$\chi_{jn} = - \int \frac{dE}{2\pi} \frac{2\Gamma_L \Gamma_R}{\mathcal{D}(0)^2} (f_L - f_R) (E - \epsilon_0), \quad (32a)$$

$$\chi_{nn} = 4\Gamma \int \frac{dE}{2\pi} \frac{E - \epsilon_0}{\mathcal{D}(0)^2} f_{eff}, \quad (32b)$$

$$S_{nn} = 4\Gamma^2 \int \frac{dE}{2\pi} \frac{f_{eff}(1 - f_{eff})}{\mathcal{D}(0)^2}, \quad (32c)$$

$$S_{jn} = 2\Gamma_L \Gamma_R \Gamma \int \frac{dE}{2\pi} \frac{1 - 2f_{eff}}{\mathcal{D}(0)^2} (f_L - f_R). \quad (32d)$$

Now it is convenient to have the integrals in Eqs.(23a) – (23f) expressed in terms of the above response and correction functions, and hence $\rho^{-(1)}$ and $\rho^{+(1)}$ in terms of χ_{jn} , χ_{nn} , S_{jn} and S_{nn} :

$$\rho^{-(1)} + \rho^{+(1)} = \frac{S_{jn}}{1 - U\chi_{nn}} + \frac{2U\chi_{jn}S_{nn}}{(1 - U\chi_{nn})^2} \quad (33)$$

$$\rho^{-(1)} - \rho^{+(1)} = \frac{-i\chi_{jn}}{1 - U\chi_{nn}}. \quad (34)$$

Substituting the Eq.(33) and (34) into Eq.(30) yields

$$S_c = 4 \left[\chi_{jn} \frac{U}{1 - U\chi_{nn}} S_{nn} \frac{U}{1 - U\chi_{nn}} \chi_{jn} + S_{jn} \frac{U}{1 - U\chi_{nn}} \chi_{jn} \right]. \quad (35)$$

Comparing this S_c expression with the previous result of S. Hershfield, we believe the constant factor 2 before the second term on the right hand side of the equation in his result is not necessary. From physical point of view, obviously, this vertex correction part of the shot noise is totally related to a set of response and correction functions of the electron density and the current. On the other hand, from technical point of view, the correction term corresponds to the vertex correction to the non-vanishing connection part in the conventional Feynman diagram expansion technique⁴². Until now, we have obtain all the important equations we are interested in for the case of single quantum dot system.

C. Results and Discussion

For each set of chosen parameters, from the self-consistent equations (20) – (22), we obtain $\rho^{(0)}$, $\rho^{-(1)}$ and $\rho^{+(1)}$, which are then substituted in Eqs.(26), (29) and (30) to compute the mean-field result of zero-frequency shot noise and its correction part. In the calculations, we set the Fermi energy zero, $E_F = 0$ in equilibrium situation, and apply the bias voltage symmetrically, $\mu_{L,R} = \pm \frac{1}{2}V$. Without loss of generality, we assume $E_0 = 0$, $U = 2\Gamma_0$ and $T = 0.1\Gamma_0$ in the following discussion concerning the symmetric case and nonsymmetric case, where Γ_0 is the energy unit during numerical calculations through out this paper

For the symmetric case, Fig.1(a) plots $\rho^{(0)}$ and $|\rho^{(1)}|$ versus the external bias voltage (note $\rho^{-(1)} = \rho^{+(1)*}$). Obviously, the magnitude of $|\rho^{(1)}|$ is one order smaller than $\rho^{(0)}$ and the peak of $|\rho^{(1)}|$ corresponds to the large increasing rate of $\rho^{(0)}$ with respect to bias voltage. The first feature indicates small S_c in comparison with S_0 , because according to the fomula of zero-frequency shot noise, a small fluctuation of electron occupation number relative to $\rho^{(0)}$ exerts a small disturbance to the mean-field result S_0 .

The second feature about the peak of S_c can be explained by the resonant tunneling effect. Actually, the Hartree approximation gives the electron energy level $\epsilon_0 = E_0 + U\rho \simeq 0.8\Gamma_0$ and hence the resonant tunneling bias voltage $1.6\Gamma_0$, if we use $\rho^{(0)} \sim 0.4$ to roughly estimate ϵ_0 at the voltage point where S_c reaches peak. It is an acceptable prediction about the tunneling voltage

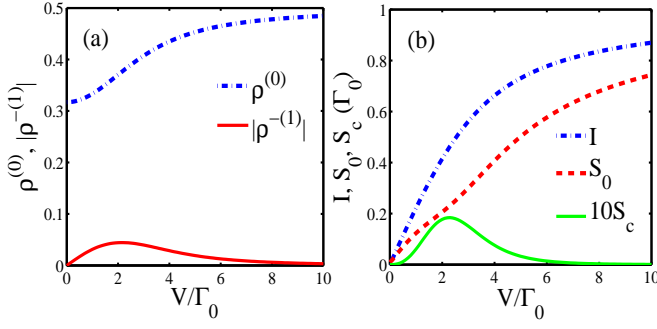


Figure 1: (Color online) (a) Zero-order and first-order of electron occupation numbers versus bias voltage. (b) Current I , noninteracting noise S_0 and 10 times correction term of shot noise $10S_c$ versus bias voltage. Parameters: $\Gamma_L = \Gamma_R = \Gamma_0$.

compared with the approximate result $2\Gamma_0$ indicated in Fig.1(a). In Fig.1(b), S_c is nearly one order of magnitude smaller than S_0 , which is consistent with the ratio of $|\rho^{(1)}|$ to $\rho^{(0)}$. In other word, the correction part of shot noise stemming from the fluctuation of electron occupation number is usually not prominent enough to increase the Fano factor largely enough and even greater than 1 (Fano factor γ is defined as $\gamma = S/(2I)$).

For nonsymmetric cases, we define the nonsymmetric degree. As illuminated in Fig.2, we plot the mean-field result term and the correction term of zero-frequency shot noise for $\gamma_{ns} = 3$ and $\gamma_{ns} = \frac{1}{3}$ respectively. In both cases, the magnitude of S_c is one order smaller than S_0 , as in the symmetric case. However, for nonsymmetric case we find that the correction term from the intradot Coulomb interaction contributes positively to the whole shot noise for $\gamma_{ns} = \frac{1}{3}$, while for $\gamma_{ns} = 3$ the contribution turns out to be negative. To interpret the sign of S_c , we would like to consider two extreme cases in the following discussions, i.e. $\gamma_{ns} \gg 1$ and $\gamma_{ns} \ll 1$. In the case of $\gamma_{ns} \gg 1$, electrons flush into the dot from the left electrode but have little chance to be pumped out into the right lead, which results in the saturation of electron occupation inside the dot and a high potential “wall” (Coulomb blockade) in the dot. This strongly strengthened Coulomb blockade immediately reduces some possibilities of dynamic processes. For example, because of the strong Coulomb repulsion in the dot, the electrons are not as free to hop from the right lead back to the dot as they do if there are no Coulomb interaction in the dot, which actually decreases the dynamic fluctuation of current. That is to say the interplay of Coulomb repulsion and many enough electrons pilling in the dot gives rise to the negative sign of S_c .

On the contrary, similar analyses for the case of $\gamma_{ns} \ll 1$ show that Coulomb interaction leads to the increase of shot noise if few electrons occupy the dot. Thus, we speculate that the sign of S_c is related to electron occupation numbers, which is associated with γ_{ns} , and the absolute contribution of S_c may goes to nearly zero if a

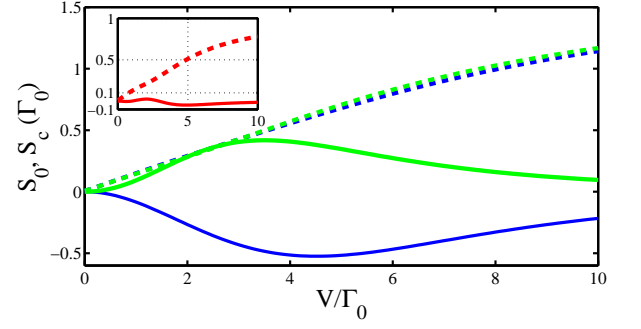


Figure 2: (Color online) Noninteracting noise S_0 (dash) and 10 times correction term of shot noise $10S_c$ (solid) versus bias voltage with $E_0 = 0$, $U = 2\Gamma_0$ and $T = 0.1\Gamma_0$ for $\gamma_{ns} = 3$ (blue), $\gamma_{ns} = \frac{1}{3}$ (green). Inset: S_0 and $10S_c$ are plotted for $\gamma_{ns} = 1.15$ (red).

proper value of γ_{ns} is chosen. For instance, the inset in Fig.2 illustrates the case of $\gamma_{ns} = 1.15$: the maximum absolute value of $10S_c$ becomes one order of magnitude smaller, being 0.048.

III. COUPLED QUANTUM DOTS

A. Model and Approximation

We employ the two-impurity Anderson model to describe the system of coupled quantum dots (CQD), i.e. $H = H_L + H_R + H_D + H_T$, in which

$$H_\eta = \sum_{\eta k} \epsilon_{\eta k} c_{\eta k}^\dagger c_{\eta k}, \quad (36)$$

$$H_D = \epsilon_1 c_1^\dagger c_1 + \epsilon_2 c_2^\dagger c_2 + \Omega(c_1^\dagger c_2 + c_2^\dagger c_1) + U c_1^\dagger c_1 c_2^\dagger c_2, \quad (37)$$

$$H_T = \sum_k \left[V_L e^{i\lambda_L(t)/2} c_1^\dagger c_{Lk} + V_R e^{i\lambda_R(t)/2} c_{Rk}^\dagger c_2 + H.c. \right]. \quad (38)$$

H_D models the coupled quantum dots with $\epsilon_1(\epsilon_2)$ denoting the energy level of the quantum dot 1(2) and Ω indicating the magnitude of electron tunneling between the two dots, where we employ the single energy level approximation. As we just want to focus our attention on the effect of interdot Coulomb interaction U in this case, we assume the Coulomb interaction in either single quantum dot goes to infinity and thus the spin index is omitted. H_T denotes the tunneling between the reservoirs and two QDs, in which the parameter V_η ($\eta = L, R$) is the corresponding tunneling magnitude between the dot and the electrode, and the measuring field (λ_η) is introduced in a symmetric way. For the convenience of further discussion, let $\lambda_{\eta-} = -\lambda_{\eta+} = \lambda_\eta$ and the symmetric configuration of external counting field requires that $\lambda_L = -\lambda_R = \lambda$.

In this case, we adopt the conventional HF approximation, which was suggested to be more reasonable than Hubbard-I approximation as long as the applied bias voltage is not relatively large⁴³, i.e.

$$c_1^\dagger c_1 c_2^\dagger c_2 \simeq \langle c_1^\dagger c_1 \rangle c_2^\dagger c_2 + c_1^\dagger c_1 \langle c_2^\dagger c_2 \rangle - \langle c_1^\dagger c_2 \rangle c_2^\dagger c_1 - \langle c_2^\dagger c_1 \rangle c_1^\dagger c_2. \quad (39)$$

Then we introduce the effective energy levels $\epsilon_{\alpha r} = \epsilon_\alpha + U\rho_{\beta\beta}$ and the effective coupling parameter $\Omega_{\alpha\beta} = \Omega - U\rho_{\alpha\beta}$ ($\alpha \neq \beta$) with $\rho_{\alpha\beta}$ being defined as $\rho_{\alpha\beta} = \langle c_\alpha^\dagger c_\beta \rangle$ ($\alpha, \beta = 1, 2$). Thus, we can put Eq.(37) in a concise form:

$$H_D = \epsilon_{1r} c_1^\dagger c_1 + \epsilon_{2r} c_2^\dagger c_2 + \Omega_{12} c_1^\dagger c_2 + \Omega_{21} c_2^\dagger c_1. \quad (40)$$

We still want to accentuate that depending on whether the time argument is on the forward path or the backward path on Keldysh contour, $\rho_{\alpha\beta}$ meets the obvious relation $\rho_{\alpha\beta}^- \neq \rho_{\alpha\beta}^+$, which results in $\epsilon_{\alpha r}^- \neq \epsilon_{\alpha r}^+$ and $\Omega_{\alpha\beta}^- \neq \Omega_{\alpha\beta}^+$.

B. Theoretical Formulation

As we can see from the single dot case, the to-be-solved expressions of the adiabatic potential, which play a central role in deriving the final results, counts on the solutions of a set of Green functions of the central region. So we employ EOM technique to solve these Green functions of the CQD, which are defined as

$$G_{\alpha\beta}(t, t') = -i \langle T_C c_\alpha(t) c_\beta^\dagger(t') \rangle_\lambda. \quad (41)$$

We introduce Green function of CQD in energy domain in the matrix form as

$$\hat{G} = \begin{pmatrix} G_{11}^{--} & G_{11}^{+-} & G_{12}^{--} & G_{12}^{+-} \\ G_{11}^{+-} & G_{11}^{++} & G_{12}^{+-} & G_{12}^{++} \\ G_{21}^{--} & G_{21}^{+-} & G_{22}^{--} & G_{22}^{+-} \\ G_{21}^{+-} & G_{21}^{++} & G_{22}^{+-} & G_{22}^{++} \end{pmatrix}. \quad (42)$$

Then some trivial calculations lead to the Dyson equation in the matrix form:

$$\hat{g}^{-1} \hat{G} = \hat{I} + \hat{\Sigma} \hat{G}, \quad (43)$$

where \hat{g} denotes the uncoupled QD Green function matrix and $\hat{\Sigma}$ the self-energy matrix. All the details are illustrated clearly in the Appendix B. However, the Dyson equation is not enough to obtain the Green functions, \hat{G} , because the self-consistent parameters ρ_{11} , ρ_{12} , ρ_{21} and ρ_{22} in Eq.(43), which can be evaluated by the following self-consistent equations:

$$\rho_{\alpha\beta}^- = \int \frac{dE}{2\pi i} G_{\alpha\beta}^{--}(E) e^{iE0^+}, \quad (44)$$

$$\rho_{\alpha\beta}^+ = \int \frac{dE}{2\pi i} G_{\alpha\beta}^{++}(E) e^{-iE0^+}. \quad (45)$$

By introducing a projection operator \hat{P} and integral operators $\hat{\mathcal{I}}_-$ and $\hat{\mathcal{I}}_+$, which are defined as $\hat{P} G_{\alpha\beta}^{\pm\pm} = G_{\alpha\beta}^{\pm\pm}$, $\hat{P} G_{\alpha\beta}^{\pm\mp} = 0$, $\hat{\mathcal{I}}_- = \int \frac{dE}{2\pi i} e^{iE0^+}$ and $\hat{\mathcal{I}}_+ = \int \frac{dE}{2\pi i} e^{-iE0^+}$, we can rewrite the eight self-consistent equations concisely in a matrix form,

$$\hat{\rho} = \hat{P} \hat{G}, \quad (46)$$

where the density matrix $\hat{\rho}$ is defined as

$$\hat{\rho} = \begin{pmatrix} \rho_{11}^- & 0 & \rho_{21}^- & 0 \\ 0 & \rho_{11}^+ & 0 & \rho_{21}^+ \\ \rho_{12}^- & 0 & \rho_{22}^- & 0 \\ 0 & \rho_{12}^+ & 0 & \rho_{22}^+ \end{pmatrix}, \quad (47)$$

and the operator matrix $\hat{\mathcal{P}}$ is defined as

$$\mathcal{P} = \begin{pmatrix} \hat{\mathcal{I}}_- \hat{P} & & & \\ & \hat{\mathcal{I}}_+ \hat{P} & & 0 \\ & & \hat{\mathcal{I}}_- \hat{P} & \\ 0 & & & \hat{\mathcal{I}}_+ \hat{P} \end{pmatrix}. \quad (48)$$

Because only the current and shot noise are considered in this work, we only need to expand $\hat{\rho}$ to the zero-order and the first-order with respect to the counting field parameters λ_L and λ_R :⁴¹

$$\hat{\rho} = \hat{\rho}^{(0)} + \sum_\eta \hat{\rho}_\eta^{(1)} \left(\frac{i}{2} \lambda_\eta \right) + o(\lambda_L, \lambda_R), \quad (49)$$

where the zero-order term $\hat{\rho}^{(0)}$ gives

$$\hat{\rho}^{(0)} = \begin{pmatrix} \rho_{11}^{(0)} & 0 & \rho_{21}^{(0)} & 0 \\ 0 & \rho_{11}^{(0)} & 0 & \rho_{21}^{(0)} \\ \rho_{12}^{(0)} & 0 & \rho_{22}^{(0)} & 0 \\ 0 & \rho_{12}^{(0)} & 0 & \rho_{22}^{(0)} \end{pmatrix}, \quad (50)$$

and the first-order term $\hat{\rho}_\eta^{(1)}$ gives

$$\hat{\rho}_\eta^{(1)} = \begin{pmatrix} \rho_{11,\eta}^{-(1)} & 0 & \rho_{21,\eta}^{-(1)} & 0 \\ 0 & \rho_{11,\eta}^{+(1)} & 0 & \rho_{21,\eta}^{+(1)} \\ \rho_{12,\eta}^{-(1)} & 0 & \rho_{22,\eta}^{-(1)} & 0 \\ 0 & \rho_{12,\eta}^{+(1)} & 0 & \rho_{22,\eta}^{+(1)} \end{pmatrix}. \quad (51)$$

Correspondingly, we expand \hat{g}^{-1} , \hat{G} and $\hat{\Sigma}$ to the first-order with respect to λ_L and λ_R :

$$\hat{g}^{-1} = \hat{g}^{-1(0)} + \sum_\eta \hat{g}_\eta^{-1(1)} \left(\frac{i}{2} \lambda_\eta \right) + o(\lambda_L, \lambda_R), \quad (52)$$

$$\hat{G} = \hat{G}^{(0)} + \sum_\eta \hat{G}_\eta^{(1)} \left(\frac{i}{2} \lambda_\eta \right) + o(\lambda_L, \lambda_R), \quad (53)$$

$$\hat{\Sigma} = \hat{\Sigma}^{(0)} + \sum_\eta \hat{\Sigma}_\eta^{(1)} \left(\frac{i}{2} \lambda_\eta \right) + o(\lambda_L, \lambda_R), \quad (54)$$

where the specific elements in the above matrixes are elaborated in Appendix B. Substituting Eqs.(52)-(54) into Eq.(43), one obtains the formal solutions of the zero-order and the first-order terms of \hat{G} in terms of \hat{g} and $\hat{\Sigma}$:

$$\hat{G}^{(0)} = (\hat{g}^{-1(0)} - \hat{\Sigma}^{(0)})^{-1}, \quad (55)$$

$$\hat{G}_L^{(1)} = \hat{G}^{(0)}(\hat{\Sigma}_L^{(1)} - \hat{g}_L^{-1(1)})\hat{G}^{(0)}, \quad (56)$$

$$\hat{G}_R^{(1)} = \hat{G}^{(0)}(\hat{\Sigma}_R^{(1)} - \hat{g}_R^{-1(1)})\hat{G}^{(0)}. \quad (57)$$

the elements of $\hat{\rho}^{(0)}$ and that of $\hat{g}_\eta^{-1(1)}$ contains the elements of $\hat{\rho}^{(0)}$ and $\hat{\rho}_\eta^{(1)}$. Inserting the expansions of $\hat{\rho}$ and \hat{G} into Eq.(46), one immediately arrive at:

$$\hat{\rho}^{(0)} = \hat{\mathcal{P}}\hat{G}^{(0)}, \quad (58)$$

$$\hat{\rho}_L^{(1)} = \hat{\mathcal{P}}\hat{G}_L^{(1)}, \quad (59)$$

$$\hat{\rho}_R^{(1)} = \hat{\mathcal{P}}\hat{G}_R^{(1)}. \quad (60)$$

Note that Eq.(B13) tells the expression of $\hat{g}^{-1(0)}$ contains

Now we are at a proper position to have $\hat{\rho}^0$ and $\hat{\rho}_\eta^{(1)}$ truly solved. Eq.(55) tells that all the elements of $\hat{G}^{(0)}$ are regarded as the function of the matrix $\hat{\rho}^{(0)}$, on the other hand $\hat{\rho}^{(0)}$ is determined by $\hat{G}^{(0)}$ as Eq.(58) indicates. Thus, combining the two matrix equations, we give the following equations to evaluate $\hat{\rho}^0$ self-consistently:

$$\rho_{11}^{(0)} = \int \frac{dE}{2\pi} \frac{1}{\Delta^{(0)}} \left\{ \Gamma_L f_L \left[(E - \epsilon_2 - U\rho_{11}^{(0)})^2 + \frac{1}{4}\Gamma_R^2 \right] + \Gamma_R f_R (\Omega - U\rho_{12}^{(0)})(\Omega - U\rho_{21}^{(0)}) \right\}, \quad (61a)$$

$$\rho_{22}^{(0)} = \int \frac{dE}{2\pi} \frac{1}{\Delta^{(0)}} \left\{ \Gamma_R f_R \left[(E - \epsilon_1 - U\rho_{22}^{(0)})^2 + \frac{1}{4}\Gamma_L^2 \right] + \Gamma_L f_L (\Omega - U\rho_{12}^{(0)})(\Omega - U\rho_{21}^{(0)}) \right\}, \quad (61b)$$

$$\rho_{21}^{(0)} = \int \frac{dE}{2\pi i} \frac{1}{\Delta^{(0)}} (\Omega - U\rho_{21}^{(0)}) \left\{ i\Gamma_L f_L (E - \epsilon_2 - U\rho_{11}^{(0)}) + i\Gamma_R f_R (E - \epsilon_1 - U\rho_{22}^{(0)}) - \frac{1}{2}\Gamma_L \Gamma_R (f_L - f_R) \right\}, \quad (61c)$$

$$\rho_{12}^{(0)} = \int \frac{dE}{2\pi i} \frac{1}{\Delta^{(0)}} (\Omega - U\rho_{12}^{(0)}) \left\{ i\Gamma_L f_L (E - \epsilon_2 - U\rho_{11}^{(0)}) + i\Gamma_R f_R (E - \epsilon_1 - U\rho_{22}^{(0)}) + \frac{1}{2}\Gamma_L \Gamma_R (f_L - f_R) \right\}. \quad (61d)$$

Note that these zero-order electron occupation numbers hold the following relations: $\rho_{11}^{(0)*} = \rho_{11}^{(0)}$, $\rho_{22}^{(0)*} = \rho_{22}^{(0)}$ and $\rho_{12}^{(0)*} = \rho_{21}^{(0)*}$. With the solved $\hat{\rho}^{(0)}$ and $\hat{G}^{(0)}$, we turn to the solution of $\hat{\rho}_L^{(1)}$. Plugging Eq.(56) into Eq.(59) yields the following linear algebraic equations:

$$\hat{\rho}_L^{(1)} = \hat{\mathcal{P}}\hat{G}^{(0)} \begin{pmatrix} 0 & -i\Gamma_L f_L & & \\ -i\Gamma_L(1-f_L) & 0 & & \\ & & 0 & 0 \\ & & 0 & 0 \end{pmatrix} \hat{G}^{(0)} - U\hat{\mathcal{P}}\hat{G}^{(0)} \begin{pmatrix} -\rho_{22,L}^{-(1)} & 0 & \rho_{21,L}^{-(1)} & 0 \\ 0 & \rho_{22,L}^{+(1)} & 0 & -\rho_{21,L}^{+(1)} \\ \rho_{12,L}^{-(1)} & 0 & -\rho_{11,L}^{-(1)} & 0 \\ 0 & -\rho_{12,L}^{+(1)} & 0 & \rho_{11,L}^{+(1)} \end{pmatrix} \hat{G}^{(0)}. \quad (62)$$

Note that similar linear algebraic equations for $\hat{\rho}_R^{(1)}$ can be reached.

All the preparations above pave for the discussion of the interested physics quantity, whose expressions will be derived in the rest of this section. The adiabatic potential is always the starting point according to the technique of FCS. According to nonequilibrium Feynman-Hellmann theorem, we have

$$\begin{aligned} \frac{\partial \mathcal{U}(\lambda_-, \lambda_+)}{\partial \lambda_{L-}} &= \left\langle \frac{\partial H_T(t)}{\partial \lambda_{L-}} \right\rangle_\lambda = \frac{i}{2} \sum_k \langle V_L e^{i\lambda_{L-}/2} c_1^\dagger c_{Lk} - V_L e^{-i\lambda_{L-}/2} c_{Lk}^\dagger c_1 \rangle_\lambda \\ &= \frac{1}{2} \sum_k \left[V_L e^{i\lambda_{L-}/2} G_{Lk,1}^{--}(t, t^+) - V_L e^{-i\lambda_{L-}/2} G_{1,Lk}^{--}(t, t^+) \right]. \end{aligned} \quad (63)$$

Applying the Keldysh disentanglement to the terms $G_{Lk,1}^{--}(t, t')$ and $G_{1,Lk}^{--}(t, t')$ in the integrand of Eq.(63), we have

$$\frac{\partial \mathcal{U}(\lambda_-, \lambda_+)}{\partial \lambda_{L-}} = \frac{1}{2} \int dt_1 \left[\Sigma_L^{--}(t, t_1) G_{11}^{--}(t_1, t) - \Sigma_L^{+-}(t, t_1) G_{11}^{+-}(t_1, t) - G_{11}^{--}(t, t_1) \Sigma_L^{--}(t_1, t) + G_{11}^{+-}(t, t_1) \Sigma_L^{+-}(t_1, t) \right], \quad (64)$$

where the self-energy is defined as $\Sigma_\eta^{\mu\nu} = \sum_k V_\eta^2 e^{i(\lambda_{\eta\mu} - \lambda_{\eta\nu})/2} g_{\eta k}^{\mu\nu}$. Performing the Fourier transform and substituting the specific expressions of the corresponding self-energy and Green functions (Appendix B), one obtain

$$\frac{\partial \mathcal{U}(\lambda_-, \lambda_+)}{\partial \lambda_{L-}} = \frac{1}{2} \Gamma_L \Gamma_R \int \frac{dE}{2\pi} \frac{1}{\Delta} \left\{ (\Omega - U\rho_{12}^+) (\Omega - U\rho_{21}^-) f_R (1 - f_L) e^{-i\tilde{\lambda}/2} - (\Omega - U\rho_{12}^-) (\Omega - U\rho_{21}^+) f_L (1 - f_R) e^{i\tilde{\lambda}/2} \right\}, \quad (65)$$

with $\tilde{\lambda} = \bar{\lambda}_L - \bar{\lambda}_R$ ($\bar{\lambda}_\eta = \lambda_{\eta-} - \lambda_{\eta+}$) and Δ being equal to the determinant of $(\hat{g}^{-1} - \hat{\Sigma})$. Based on the identity within the adiabatic method $\frac{\partial \ln \chi}{\partial (\lambda_L/2)} = -i\mathcal{T} \frac{\partial \mathcal{U}}{\partial (\lambda_L/2)}$, the current is given as

$$I = \frac{1}{\mathcal{T}} \frac{\partial \ln \chi}{\partial (i\lambda_L/2)} \Big|_{\lambda=0} = \int \frac{dE}{2\pi} T_0(E) (f_L - f_R), \quad (66)$$

where the zero-order transmission coefficient $T_0(E)$ is defined as

$$T_0(E) = \Gamma_L \Gamma_R (\Omega - U\rho_{12}^{(0)}) (\Omega - U\rho_{21}^{(0)}) \left| (E - \epsilon_1 - U\rho_{22}^{(0)} + \frac{i}{2}\Gamma_L)(E - \epsilon_2 - U\rho_{11}^{(0)} + \frac{i}{2}\Gamma_R) - (\Omega - U\rho_{12}^{(0)})(\Omega - U\rho_{12}^{(0)}) \right|^{-2}. \quad (67)$$

When it comes to the expression of the zero-frequency shot noise, we first rewrite the formula for the adiabatic potential as

$$\frac{\partial \mathcal{U}}{\partial (\lambda_L/2)} = \int \frac{dE}{2\pi} \frac{\left[T_\lambda^{(1)}(E) f_R (1 - f_L) e^{-i\tilde{\lambda}/2} - T_\lambda^{(2)}(E) f_L (1 - f_R) e^{i\tilde{\lambda}/2} \right]}{1 + \left[T_\lambda^{(1)}(E) f_R (1 - f_L) (e^{-i\tilde{\lambda}/2} - 1) + T_\lambda^{(2)}(E) f_L (1 - f_R) (e^{i\tilde{\lambda}/2} - 1) \right]}, \quad (68)$$

where we define the transmission coefficients $T_\lambda^{(1)}(E) = [\Gamma_L \Gamma_R (\Omega - U\rho_{12}^+) (\Omega - U\rho_{21}^-)] / \Delta_i$ and $T_\lambda^{(2)}(E) = [\Gamma_L \Gamma_R (\Omega - U\rho_{12}^-) (\Omega - U\rho_{21}^+)] / \Delta_i$ (refer to Eq.(B8) in Appendix B for Δ_i), which hold the relation $T_\lambda^{(1)}(E)|_{\lambda=0} = T_\lambda^{(2)}(E)|_{\lambda=0} = T_0(E)$. In this way, we make it technically natural to separate the shot noise into two parts, i.e.

$$S = 2 \frac{1}{\mathcal{T}} \frac{\partial \ln^2 \chi}{\partial (i\lambda_L/2)^2} \Big|_{\lambda=0} = S^{(0)} + S^{(c)}, \quad (69)$$

where $S^{(0)}$ and $S^{(c)}$ are respectively the mean-field result and the correction part from the interdot Coulomb interaction. Actually, $S^{(0)}$ holds the same form as the single QD case, and is derived as

$$S^{(0)} = 2 \int \frac{dE}{2\pi} \{ T_0(E) [f_R (1 - f_L) + f_L (1 - f_R)] - T_0^2(E) (f_L - f_R)^2 \}. \quad (70)$$

On the contrary, the explicit expression of $S^{(c)}$ is different from that of the single QD:

$$S^{(c)} = 2 \int \frac{dE}{2\pi} \left[\frac{\partial T_\lambda^{(2)}(E)}{\partial (i\lambda_L/2)} \Big|_{\lambda=0} f_L (1 - f_R) - \frac{\partial T_\lambda^{(1)}(E)}{\partial (i\lambda_L/2)} \Big|_{\lambda=0} f_R (1 - f_L) \right]. \quad (71)$$

Writing out the derivative of $T_\lambda(E)$ with respect to $(i\lambda_L/2)$, we find that $S^{(c)}$ can be split into two parts $S_1^{(c)}$ and $S_2^{(c)}$ corresponding respectively to the fluctuations of the electronic occupation numbers, $\rho_{\beta\beta}$, and coherence terms, $\rho_{\alpha\beta}$ ($\alpha \neq \beta$). We might as well define some auxiliary quantity to make their expressions more succinct: $\tilde{\epsilon}_1^{(0)} = E - \epsilon_1 - U\rho_{22}^{(0)}$, $\tilde{\epsilon}_2^{(0)} = E - \epsilon_2 - U\rho_{11}^{(0)}$, $\Omega_{12}^{(0)} = \Omega - U\rho_{12}^{(0)}$ and $\Omega_{21}^{(0)} = \Omega - U\rho_{21}^{(0)}$. Then we give the formula for $S_1^{(c)}$ as

$$S_1^{(c)} = \frac{2U}{\Gamma_L \Gamma_R} \int \frac{dE}{2\pi} T_0^2(E) \left\{ \Omega_{12}^{(0)} \Omega_{21}^{(0)} (f_L - f_R) \left[\mathcal{K}_1 (\rho_{22,L}^{-(1)} + \rho_{22,L}^{+(1)}) + \mathcal{K}_2 (\rho_{11,L}^{-(1)} + \rho_{11,L}^{+(1)}) + \mathcal{K}_3 (\rho_{22,L}^{-(1)} - \rho_{22,L}^{+(1)}) + \mathcal{K}_4 (\rho_{11,L}^{-(1)} - \rho_{11,L}^{+(1)}) \right] \right\}, \quad (72)$$

with

$$\mathcal{K}_1 = \tilde{\epsilon}_1^{(0)} \left[(\tilde{\epsilon}_2^{(0)})^2 + \frac{1}{4} \Gamma_R^2 \right] - \tilde{\epsilon}_2^{(0)} \Omega_{12}^{(0)} \Omega_{21}^{(0)}, \quad (73a)$$

$$\mathcal{K}_2 = \tilde{\epsilon}_2^{(0)} \left[(\tilde{\epsilon}_1^{(0)})^2 + \frac{1}{4} \Gamma_L^2 \right] - \tilde{\epsilon}_1^{(0)} \Omega_{12}^{(0)} \Omega_{21}^{(0)}, \quad (73b)$$

$$\mathcal{K}_3 = i\Gamma_L \left(f_L - \frac{1}{2} \right) \left[(\tilde{\epsilon}_2^{(0)})^2 + \frac{1}{4} \Gamma_R^2 \right] - i\Gamma_R (f_R - \frac{1}{2}) \Omega_{12}^{(0)} \Omega_{21}^{(0)}, \quad (73c)$$

$$\mathcal{K}_4 = i\Gamma_R \left(f_R - \frac{1}{2} \right) \left[(\tilde{\epsilon}_1^{(0)})^2 + \frac{1}{4}\Gamma_L^2 \right] - i\Gamma_L (f_L - \frac{1}{2}) \Omega_{12}^{(0)} \Omega_{21}^{(0)}, \quad (73d)$$

in which the fluctuations of electron occupation numbers, or the first-order terms in the expansion of ρ_{11} and ρ_{22} i.e. $\rho_{11,\eta}^{\mu(1)}$ and $\rho_{22,\eta}^{\mu(1)}$, are involved. For $S_2^{(c)}$, we have

$$\begin{aligned} S_2^{(c)} = 2U \int \frac{dE}{2\pi} \left\{ -\frac{1}{\Gamma_L \Gamma_R} T_0^2(E) \Omega_{12}^{(0)} \Omega_{21}^{(0)} (f_L - f_R) \left[\mathcal{L}_1 (\rho_{12,L}^{-(1)} + \rho_{12,L}^{+(1)}) + \mathcal{L}_2 (\rho_{21,L}^{-(1)} + \rho_{21,L}^{+(1)}) \right. \right. \\ \left. \left. + \mathcal{L}_3 (\rho_{12,L}^{-(1)} - \rho_{12,L}^{+(1)}) + \mathcal{L}_4 (\rho_{21,L}^{-(1)} - \rho_{21,L}^{+(1)}) \right] + \Omega_{21}^{(0)} T_0(E) \left[\rho_{12,L}^{+(1)} f_R (1 - f_L) - \rho_{12,L}^{-(1)} f_L (1 - f_R) \right] \right. \\ \left. + \Omega_{12}^{(0)} T_0(E) \left[\rho_{21,L}^{-(1)} f_R (1 - f_L) - \rho_{21,L}^{+(1)} f_L (1 - f_R) \right] \right\}, \end{aligned} \quad (74)$$

with

$$\mathcal{L}_1 = \Omega_{21}^{(0)} (\tilde{\epsilon}_1^{(0)} \tilde{\epsilon}_2^{(0)} - \Omega_{12}^{(0)} \Omega_{21}^{(0)} - \frac{1}{4} \Gamma_L \Gamma_R) \quad (75a)$$

$$\mathcal{L}_2 = \Omega_{12}^{(0)} (\tilde{\epsilon}_1^{(0)} \tilde{\epsilon}_2^{(0)} - \Omega_{12}^{(0)} \Omega_{21}^{(0)} - \frac{1}{4} \Gamma_L \Gamma_R), \quad (75b)$$

$$\mathcal{L}_3 = \Omega_{21}^{(0)} \left[i\Gamma_L \left(f_L - \frac{1}{2} \right) \tilde{\epsilon}_2^{(0)} + i\Gamma_R \left(f_R - \frac{1}{2} \right) \tilde{\epsilon}_1^{(0)} - \frac{1}{2} \Gamma_L \Gamma_R (f_L - f_R) \right], \quad (75c)$$

$$\mathcal{L}_4 = \Omega_{12}^{(0)} \left[i\Gamma_L \left(f_L - \frac{1}{2} \right) \tilde{\epsilon}_2^{(0)} + i\Gamma_R \left(f_R - \frac{1}{2} \right) \tilde{\epsilon}_1^{(0)} + \frac{1}{2} \Gamma_L \Gamma_R (f_L - f_R) \right], \quad (75d)$$

which is related to the first-order terms in the expansions of ρ_{12} and ρ_{21} , i.e. $\rho_{12,\eta}^{\mu(1)}$ and $\rho_{21,\eta}^{\mu(1)}$. Thus, according to which elements in the first-order terms of density matrixes ($\hat{\rho}_\eta^{(1)}$) are involved in the expression of correction part of shot noise, we divide the formula of $S^{(c)}$ into $S_1^{(c)}$ and $S_2^{(c)}$. Considering the similarity between $S_1^{(c)}$ and $S^{(c)}$, which is also related to the fluctuation of electron occupation number, we can expect the absolute value of $S_1^{(c)}$ is much smaller than $S^{(0)}$. However, different from the single dot system, in the coupled dots system the new additional correction part of zero-frequency shot noise, $S_2^{(c)}$, stems from the coherence dynamic fluctuations ($\rho_{12,\eta}^{\mu(1)}$ and $\rho_{21,\eta}^{\mu(1)}$), which can play a dominating role in increasing shot noise and Fano factor (γ), and can even give rise to $\gamma > 1$ under certain circumstances, as illustrated in the following section.

C. Results and Discussion

All the elements of $\hat{\rho}^{(0)}$ and $\hat{\rho}_\eta^{(1)}$ are computed from self-consistent Eqs. (61a)–(62) and then are used to evaluate the current and the zero-frequency shot noise. In the following discussions, the correction part of shot noise resulting from the coherence fluctuations are highlighted to find the parameter window for a large Fano factor γ , especially for $\gamma > 1$.

We consider a typical symmetric case with $\Gamma_L = \Gamma_R = \Gamma_0$ and $\epsilon_1 = \epsilon_2 = \Gamma_0$, and thereby plot the calculated results in Fig. 3. In Fig. 3(a), the electron occupation numbers $\rho_{11}^{(0)}$ and $\rho_{22}^{(0)}$ are plotted as functions of the bias voltage, together with the bias dependence of $|\rho_{22}^{-(1)}|$ that reflects the nonequilibrium density fluctuation (NDF) of ρ_{22} . In Fig. 3(b), we plot the coherence terms, ($|\rho_{12}^{(0)}|$, $|\rho_{21}^{(0)}|$), and the nonequilibrium coherence fluctuations (NCFs), ($|\rho_{12}^{-(1)}|$, $|\rho_{21}^{-(1)}|$) as functions of the

bias voltage. In Fig. 3(c), $S_1^{(c)}$ and $S_2^{(c)}$ versus the bias voltage are depicted. From these figures, we find three features in the symmetric case: (1) The ratio of $|\rho_{12}^{-(1)}|$ ($|\rho_{21}^{-(1)}|$) to $\rho_{12}^{(0)}$ ($\rho_{21}^{(0)}$) is much larger than the ratio of $|\rho_{22}^{-(1)}|$ to $\rho_{22}^{(0)}$ at nearly whole regions of the bias voltages, except for the region of relatively small bias voltages; (2) The second correction term, $S_2^{(c)}$, causes the main contribution to the shot noise of a CQD, other than the first correction term, $S_1^{(c)}$. As far as the resulted formulae of $S_1^{(c)}$ and $S_2^{(c)}$, i.e., Eqs. (72) and (74) in the above subsection, are concerned, this feature can be ascribed to the first feature from numerical calculation point of view. In consequence, we can thereby deduce that the NCF effect plays a more prominent role in determining the zero-frequency shot noise of a CQD system; (3) $|\rho_{22}^{-(1)}|$, $|\rho_{12}^{-(1)}|$, $|\rho_{21}^{-(1)}|$, $S_2^{(c)}$ and S all exhibit the peak and the plateau structures respectively with increasing bias voltage. The peaks are located at the value

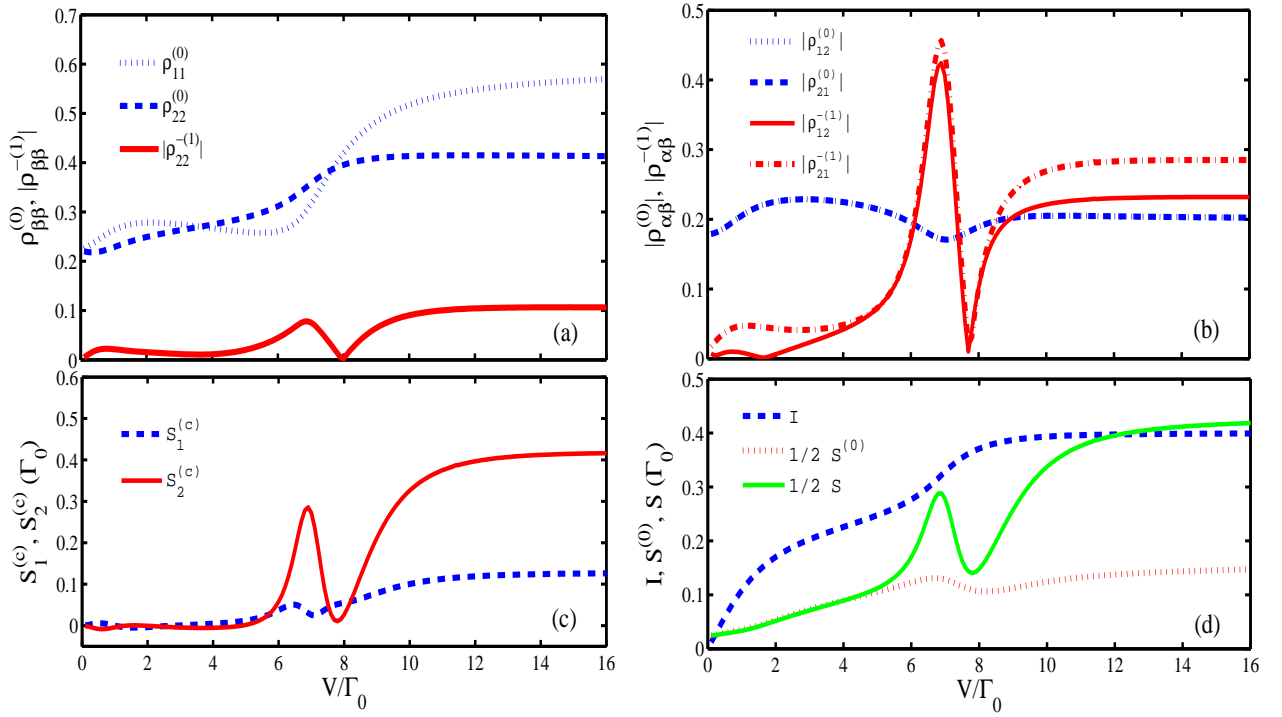


Figure 3: (Color online) (a) Zero-order terms ($\rho_{\beta\beta}^{(0)}$) and first-order terms ($|\rho_{\beta\beta}^{-(1)}|$) of electron occupation numbers versus bias voltage; (b) Zero-order coherence terms ($|\rho_{\alpha\beta}^{(0)}|$) and first-order coherence terms ($|\rho_{\alpha\beta}^{-(1)}|$) versus bias voltage, where we let $\alpha \neq \beta$ here; (c) Two separate components $S_1^{(c)}$ and $S_2^{(c)}$ of the shot noise correction terms versus bias voltage; (d) Current I , noninteracting noise $S^{(0)}$ and total shot noise S versus bias voltage. Other Parameters used in the calculations: $\epsilon_1 = \epsilon_2 = \Gamma_0$, $\Gamma_L = \Gamma_R = \Gamma_0$, $\Omega = \Gamma_0$, $U = 4\Gamma_0$, and $T = 0.1\Gamma_0$.

of bias voltage where a large change rate of electron occupation number takes place, being around $V = 7\Gamma_0$, which can be explained by one of the approximate eigenvalues $\epsilon_{\alpha r} + \Omega_{\alpha\beta}$. While the plateaus appear when the bias voltage becomes greater than $V = 2(\epsilon_\alpha + U) = 10\Gamma_0$. In addition, Eq.(74) requires that $S_2^{(c)}$ and $|\rho_{\alpha\beta}^{-(1)}|$ share the two structures nearly at the same values of bias voltages. To understand such connections among $|\rho_{22}^{-(1)}|$, $|\rho_{12}^{-(1)}|$ and $|\rho_{21}^{-(1)}|$, one needs to notice the HF energy levels for the two dots are $\epsilon_\alpha + U\rho_{\beta\beta}$ ($\alpha \neq \beta$). That is to say that the energy level of the dot α becomes uncertain at nonequilibrium condition and fluctuates roughly between $\epsilon_\alpha + U(\rho_{\beta\beta}^{(0)} \pm |\rho_{\beta\beta}^{-(1)}|)$ due to the NDF effect. Its average energy level is $\epsilon_\alpha + U\rho_{\beta\beta}^{(0)}$. Since the gap between energy levels of two QDs directly affects the overlap between the two electronic wavefunctions of the two dots, the energy level fluctuations induced by NDFs naturally result in the NCFs, which accounts for the reason that the peak and plateau structures appear almost at the same values of bias voltages for $|\rho_{22}^{-(1)}|$, $|\rho_{12}^{-(1)}|$ and $|\rho_{21}^{-(1)}|$. Surprisingly, a weak super-Poissonian shot noise is found from Fig. 3(d) at the considerably large bias voltage region, $V > 12\Gamma_0$, which is different from previous results based on the quantum rate equation (QRE).^{12,23,24,31} These earlier work predict no super-Poissonian noise for the

symmetric CQD in the limit of infinite bias voltage at zero temperature. Notice that the QRE approach is only valid for sequential tunneling and thereby fails to account for quantum coherence between two dots. Meanwhile, notice that the enhancement of shot noise is mainly coming from the contribution of $S_2^{(c)}$, i.e., the NCFs, in the present studies. We can thereby argue that the coherent exchange interaction effect is somehow accounted in certain degree in the present calculations of current fluctuations at the level of HF approximation, and causes the consequence of the enhanced noise.

In order to find out the optimal parameter values for the super-Poissonian noise ($\gamma > 1$), it is necessary in the following to investigate the two structures more deeply. On the premise of keeping the ratio $U/\Omega = 4$ unchanged, we first study another two symmetric systems, i.e. (i) $U = 2$, $\Omega = 0.5$ and (ii) $U = 5$, $\Omega = 1.25$, and depict their results in Figs. 4(a,c) and Figs.4(b,d), respectively. For the sake of convenience, we temporarily name the previous system of $U = 4$ and $\Omega = 1$ as the system (iii). For the systems (ii) and (iii), the peak of $|\rho_{22}^{-(1)}|$ corresponds to the large slopes or gradients of $\rho_{22}^{(0)}$ with respect to the bias voltage, while the peak disappears in the system (i) although there is still a large slope for $\rho_{22}^{(0)}$ as shown in Fig. 4(a). To figure out why the NCFs show different behaviors in these systems, we notice that the

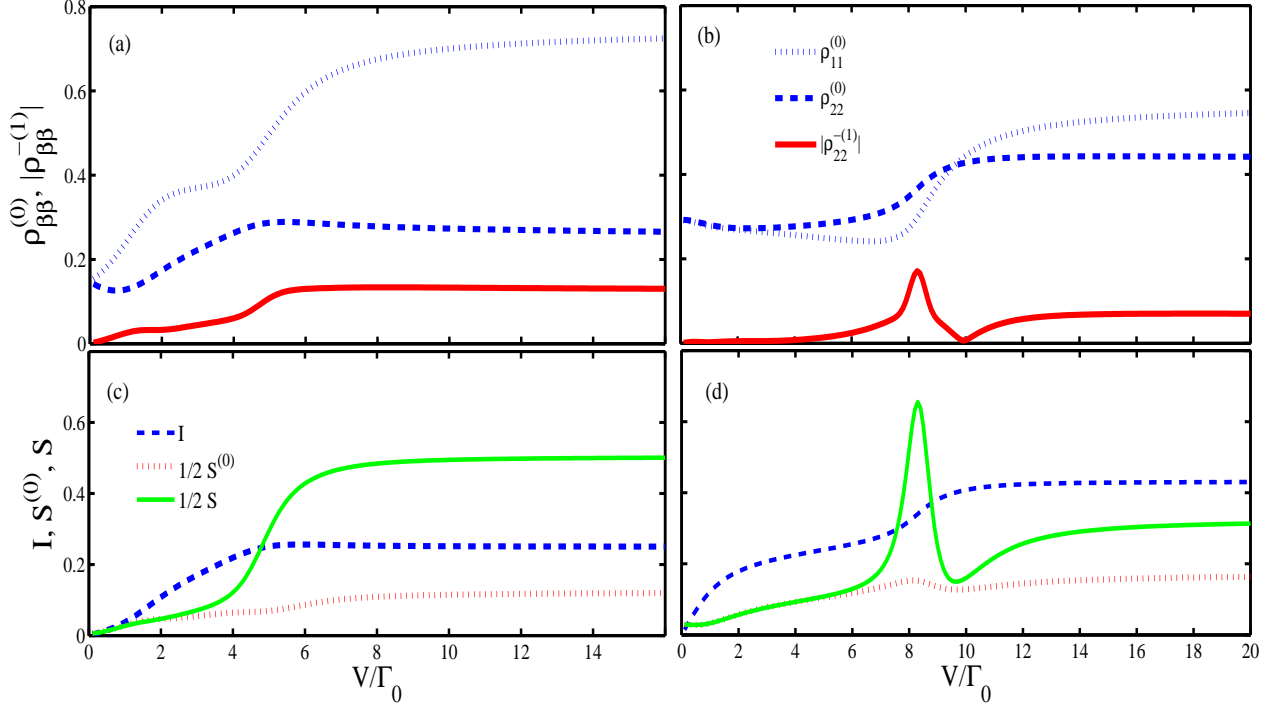


Figure 4: (Color online) Zero-order terms ($\rho_{\beta\beta}^{(0)}$) and first-order terms ($|\rho_{\beta\beta}^{-(1)}|$) of electron occupation numbers versus bias voltage in (a, b), and current I , noninteracting noise $S^{(0)}$ and total shot noise S versus bias voltage in (c, d). The case of $\Omega = 0.5\Gamma_0$ and $U = 2\Gamma_0$ is plotted in (a, c) and the case of $\Omega = 1.25\Gamma_0$ and $U = 5\Gamma_0$ in (b, d). Other Parameters are the same as those in Fig. 3.

interdot coupling parameter Ω of the systems (ii) and (iii) is much larger than the system (i). It is understandable that a larger ratio of Ω to Γ_η means electrons being much easier to hop from dot 1 to dot 2 than electrons tunneling between the left lead and dot 1 or between the right lead and dot 2. As a result, a larger Ω relative to Γ_η can narrow down the difference between $\rho_{11}^{(0)}$ and $\rho_{22}^{(0)}$ and hence the gap between two renormalized energy levels. Since a smaller energy gap implies stronger electronic wavefunction overlap between two dots, we can therefore ascribe the peak structure at the finite bias voltage to the stronger coherence and NCFs in the systems (ii) and (iii). Furthermore, we find that the system (ii) even shows a super-Poissonian noise due to the strong peak in the noise-voltage characteristics at a small bias region (noticing that this system has the strongest hopping $\Omega = 1.25\Gamma_0$ in our calculations). This new noise enhancement at small bias region is also different from previous prediction by Aghassi, in which a thermally excited multi-electron state is involved in sequential tunneling process and thus crucial for an enhanced noisy current.²²

With regard to the plateau structure, one can observe that the plateau is always accompanied by prominent difference between $\rho_{11}^{(0)}$ and $\rho_{22}^{(0)}$ at the large bias voltage region. As we indicated above, bigger density difference will cause more stronger NCF and inevitably cause more

remarkable enhancement of the shot noise. This fact interprets the occurrence of weak super-Poissonian noise in the system (iii) at the large bias voltage region as shown in Fig. 3(d). Meanwhile, the system (i) with the smallest hopping $\Omega = 0.5\Gamma_0$ will have the most biggest density difference in the present study and thereby exhibits the most obvious super-Poissonian noise at the large bias voltage region as display in Fig. 4(c). Until now, we demonstrate that on the premise of setting U/Ω constant, a larger inter-dot hopping Ω induces a remarkable peak at small bias voltages, while a smaller hopping Ω leads to a prominent plateau structure at large bias voltages.

After these discussions with keeping U/Ω constant, we now consider different relative values of U to Ω . We plot Fano factor versus the bias voltage for different values of U with $\Omega = 1$ in Fig. 5(a) and different values of Ω with $U = 4$ in Fig. 5(b). From these results, we can argue that the Fano factor γ is, in a certain degree, positively associated with the relative ratio U/Ω . In fact, we notice that the HF approximation used in this paper defines an effective electron hopping parameter, $\Omega[1 - (U/\Omega)\rho_{\alpha\beta}]$, as indicated in the Hamiltonian Eq. (40). As a result, it seems reasonable that we can regard the ratio U/Ω as a magnifying factor of the interdot coherence ($\rho_{12}^{(0)}$ and $\rho_{21}^{(0)}$) and hence of the NCFs ($\rho_{12}^{(1)}$ and $\rho_{21}^{(1)}$). Since $S_2^{(c)}$ is positively correlated to the NCFs, $\rho_{12}^{(1)}$ and $\rho_{21}^{(1)}$, the

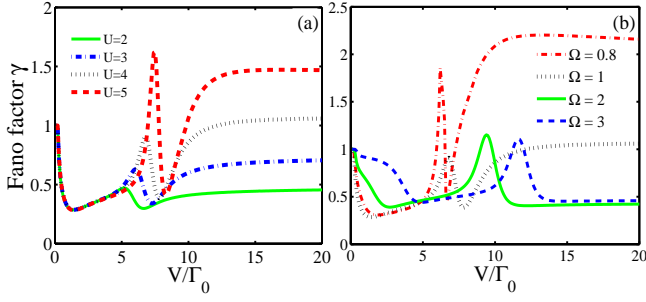


Figure 5: (Color online) Fano factor versus bias voltage for a fixed $\Omega = 1$ in (a) and a fixed $U = 4$ in (b), respectively. The other parameters are the same as those in Fig. 3.

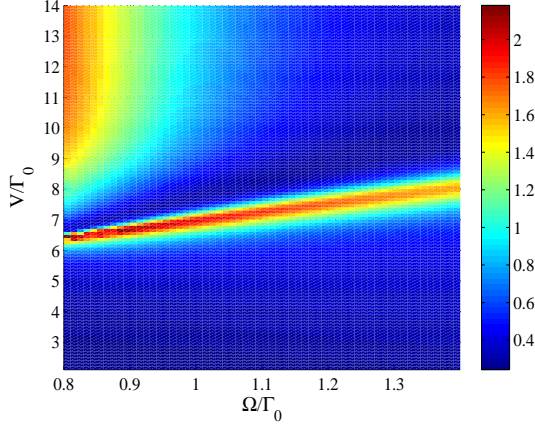


Figure 6: (Color online) Fano factor versus bias voltage V and interdot coupling parameter Ω with the following parameters: $\epsilon_1 = \epsilon_2 = \Gamma_0$, $U = 4\Gamma_0$, and $T = 0.1\Gamma_0$.

larger ratio U/Ω will naturally cause the larger $S_2^{(c)}$. It is worth noticing that this argument agrees qualitatively well with the rate-equation investigation by G. Kieflich *et.al.* on the noise enhancement for the CQD system, in which the authors ascribed the noise enhancement to the interplay of Coulomb interaction and coherence.¹² Finally, we show the Fano factor as functions of the bias voltage and the undressed hopping parameter Ω with a fixed $U = 4\Gamma_0$ in Fig. 6.

IV. CONCLUSIONS

In this paper, we have investigated the zero-frequency shot noise of electronic tunneling through a SQD and a CQD focusing our attention on the effects of the intradot or interdot Coulomb interactions, respectively, within the theoretical framework of full counting statistics. Employing NGF techniques, we have derived the analytical expressions for the current and shot noise by self-consistent solutions within the Hartree approximation in a SQD system and the HF approximation in a CQD system.

For the SQD system, the obtained explicit expression

for the correction term of zero-frequency shot noise, S_c , caused by the intradot Coulomb interaction, has been found to be in agreement with the earlier result by Hershfield using the Feynman diagram expansion method. Our numerical calculations have shown that the contribution of the correlation term is nearly negligible in comparison with the noninteracting shot noise (at least one order of magnitude smaller in our calculations for chosen parameters). In addition, it is interestingly found that altering the relative values of Γ_L to Γ_R can change the sign of S_c .

The main objective of this paper is to examine the zero-frequency shot noise in a symmetric CQD by taking account of the joint effects of Coulomb interaction and coherent exchange between two dots beyond the usual rate-equation investigations. For this purpose, we have adopted the HF approximation to deal with the interdot Coulomb interaction, which is proved to be a reliable method for electronic tunneling through a CQD as long as the applied bias voltage is not relatively large.⁴³ We have found that the correction term of the zero-frequency shot noise of a CQD can be separated into two parts: one is coming from the contribution of the NDFs and the other one is originated from the contribution of the interdot NCFs. The numerical calculations have predicted that the interdot NCFs play a predominated role in determining the shot noise over the NDFs, and give rise to two characteristic structures, a peak and a plateau, in the behavior of bias-voltage-dependent shot noise, whose values may be enhanced to show super-Poissonian even for the symmetric CQD. It is worth pointing out that this result is new and different from those of rate-equation calculations, which predicted either no super-Poissonian noise or super-Poissonian noise due to the thermal excited multi-electron state in the symmetric case. We hope that the present paper can stimulate further experimental investigation for shot noise in a CQD system.

Acknowledgments

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Appendix A: Calculation of the contour-order GFs for the SQD

In the appendix A, we employ the EOM technique to derive the contour-order GFs of the central region for the SQD system.

Considering the time evolution of $d_\sigma(t)$, one can easily obtain the equations of motion for $g_{d\sigma}^{--}(t, t')$ and $g_{d\sigma}^{++}(t, t')$ and then perform the Fourier transformation

to give

$$\hat{g}_{d\sigma}^{-1} = \begin{pmatrix} E - \epsilon_0^- & 0 \\ 0 & -(E - \epsilon_0^+) \end{pmatrix}. \quad (\text{A1})$$

Next, one can proceed in the similar way to obtain the EOMs for GFs of the interacting quantum dot:

$$\left(i\frac{\partial}{\partial t} - \epsilon_0^\mu\right) G_{d\sigma}^{\mu\nu}(t, t') = \delta_C(t - t') + \sum_{\eta k} V_{\eta k} e^{i\lambda_{\eta\mu}/2} G_{\eta k\sigma, d\sigma}^{\mu\nu}(t, t'), \quad (\text{A2})$$

where the contour order Dirac delta function $\delta_C(t - t')$ is defined as (C_- indicates the forward pathway and C_+ indicates the backward pathway on the Keldysh contour): $\delta_C(t - t') = \delta(t - t')(t, t' \in C_-)$, $\delta_C(t - t') = -\delta(t - t')(t, t' \in C_+)$ and $\delta_C(t - t') = 0$ ($t \in C_-, t' \in C_+$ or $t \in C_+, t' \in C_-$). After Applying the Keldysh disentanglement, i.e.

$$G_{\eta k\sigma, d\sigma}^{\mu\nu}(t, t') = V_{\eta k} \int dt_1 \left[g_{\eta k\sigma}^{\mu-}(t, t_1) e^{-i\lambda_{\eta-}/2} G_{d\sigma}^{-\nu}(t_1, t') - g_{\eta k\sigma}^{\mu+}(t, t_1) e^{-i\lambda_{\eta+}/2} G_{d\sigma}^{+\nu}(t_1, t') \right], \quad (\text{A3})$$

and

$$G_{d\sigma, \eta k\sigma}^{\mu\nu}(t, t') = V_{\eta k} \int dt_1 \left[G_{d\sigma}^{\mu-}(t, t_1) e^{i\lambda_{\eta-}/2} g_{\eta k\sigma}^{-\nu}(t_1, t') - G_{d\sigma}^{\mu+}(t, t_1) e^{i\lambda_{\eta+}/2} g_{\eta k\sigma}^{+\nu}(t_1, t') \right], \quad (\text{A4})$$

and the Fourier transformation, the above Green functions are reconstructed in a concise matrix form of Dyson equation $\hat{G}_{d\sigma} = \hat{g}_{d\sigma} + \hat{g}_{d\sigma} \hat{\Sigma} \hat{G}_{d\sigma}$, where $\hat{\Sigma}$ is the introduced self-energy matrix, expressed as

$$\hat{\Sigma} = \begin{pmatrix} \Sigma_{L\sigma}^{--} + \Sigma_{R\sigma}^{--} & -(\Sigma_{L\sigma}^{+-} + \Sigma_{R\sigma}^{+-}) \\ -(\Sigma_{L\sigma}^{+-} + \Sigma_{R\sigma}^{+-}) & \Sigma_{L\sigma}^{++} + \Sigma_{R\sigma}^{++} \end{pmatrix}. \quad (\text{A5})$$

Here, $\Sigma_{\eta\sigma}^{\mu\nu}$ ($\eta = L, R$) is the self-energy. The explicit expressions for $\Sigma_{\eta\sigma}^{\mu\nu}$ are listed as follows:

$$\begin{aligned} \Sigma_{L\sigma}^{--} &= i\Gamma_L(f_L - \frac{1}{2}), \quad \Sigma_{L\sigma}^{+-} = ie^{i\bar{\lambda}_L/2}\Gamma_L f_L, \quad \Sigma_{L\sigma}^{+ -} = -i\Gamma_L e^{-i\bar{\lambda}_L/2}(1 - f_L), \quad \Sigma_{L\sigma}^{++} = i\Gamma_L(f_L - \frac{1}{2}), \\ \Sigma_{R\sigma}^{--} &= i\Gamma_R(f_R - \frac{1}{2}), \quad \Sigma_{R\sigma}^{+-} = i\Gamma_R f_R, \quad \Sigma_{R\sigma}^{+ -} = -i\Gamma_R(1 - f_R), \quad \Sigma_{R\sigma}^{++} = i\Gamma_R(f_R - \frac{1}{2}). \end{aligned} \quad (\text{A6})$$

Therefore, the corresponding GFs can be told from $\hat{G}_{d\sigma} = (\hat{g}_{d\sigma}^{-1} - \hat{\Sigma})^{-1}$, which gives Eq.(14).

Appendix B: Calculations of the contour-order GFs for the CQD and its expansion with respect to the counting fields

In this appendix, we first apply the EOM technique to calculate the GFs for the CQD system, and then perform expansion of the resulted Dyson equation in to zero-order and first-order with respect to the counting fields.

We first bring in the matrix \hat{g} and construct the equations of motion for all its elements into the matrix form in energy domain as

$$\hat{g}^{-1} = \begin{pmatrix} g_{11}^{--} & 0 & g_{12}^{--} & 0 \\ 0 & g_{11}^{++} & 0 & g_{12}^{++} \\ g_{21}^{+-} & 0 & g_{22}^{+-} & 0 \\ 0 & g_{21}^{++} & 0 & g_{22}^{++} \end{pmatrix}^{-1} = \begin{pmatrix} E - \epsilon_{1r}^- & 0 & -\Omega_{12}^- & 0 \\ 0 & -(E - \epsilon_{1r}^+) & 0 & \Omega_{12}^+ \\ -\Omega_{21}^- & 0 & E - \epsilon_{2r}^- & 0 \\ 0 & \Omega_{21}^+ & 0 & -(E - \epsilon_{2r}^+) \end{pmatrix}, \quad (\text{B1})$$

Note that uncoupled QD Green functions with t and t' on the different pathways on the Keldysh contour vanish if only the final steady state is discussed. The dot Keldysh nonequilibrium Green functions meet the following equations of motion:

$$\left(i\frac{\partial}{\partial t} - \epsilon_{1r}^\mu\right) G_{11}^{\mu\nu}(t, t') = \delta_C(t - t') + \sum_k e^{i\lambda_{L\mu}/2} V_L G_{Lk,1}^{\mu\nu}(t, t') + \Omega_{12}^\mu G_{21}^{\mu\nu}(t, t'), \quad (\text{B2a})$$

$$\left(i\frac{\partial}{\partial t} - \epsilon_{2r}^\mu\right) G_{22}^{\mu\nu}(t, t') = \delta_C(t - t') + \sum_k e^{i\lambda_{R\mu}/2} V_R G_{Rk,2}^{\mu\nu}(t, t') + \Omega_{21}^\mu G_{12}^{\mu\nu}(t, t'), \quad (\text{B2b})$$

$$\left(i\frac{\partial}{\partial t} - \epsilon_{1r}^\mu\right) G_{12}^{\mu\nu}(t, t') = \sum_k e^{i\lambda_{L\mu}/2} V_L G_{Lk,2}^{\mu\nu}(t, t') + \Omega_{12}^\mu G_{22}^{\mu\nu}(t, t'), \quad (\text{B2c})$$

$$\left(i\frac{\partial}{\partial t} - \epsilon_{2r}^\mu\right) G_{21}^{\mu\nu}(t, t') = \sum_k e^{i\lambda_{R\mu}/2} V_R G_{Rk,1}^{\mu\nu}(t, t') + \Omega_{21}^\mu G_{11}^{\mu\nu}(t, t'), \quad (\text{B2d})$$

where the mixed Green functions $G_{\eta k,1}(t, t')$ and $G_{1,\eta k}(t, t')$ are defined as

$$G_{\eta k,\alpha}(t, t') = -i\langle T_C c_{\eta k}(t) c_\alpha^\dagger(t') \rangle_\lambda, \quad G_{\alpha,\eta k}(t, t') = -i\langle T_C c_\alpha(t) c_{\eta k}^\dagger(t') \rangle_\lambda. \quad (\text{B3})$$

After applying Fourier transformation, Eqs.(B2a)-(B2d) are constructed as the matrix form of Dyson equation, i.e. Eq.(43), where we introduce the self-energy matrix

$$\hat{\Sigma} = \begin{pmatrix} \Sigma_L^{--} & -\Sigma_L^{-+} & 0 & 0 \\ -\Sigma_L^{+-} & \Sigma_L^{++} & 0 & 0 \\ 0 & 0 & \Sigma_R^{--} & -\Sigma_R^{-+} \\ 0 & 0 & -\Sigma_R^{+-} & \Sigma_R^{++} \end{pmatrix}, \quad (\text{B4})$$

and all the elements of $\hat{\Sigma}$ are listed as follows:

$$\begin{aligned} \Sigma_L^{--} &= i\Gamma_L(f_L - \frac{1}{2}), \quad \Sigma_L^{-+} = ie^{i\bar{\lambda}_L/2}\Gamma_L f_L, \quad \Sigma_L^{+-} = -ie^{-i\bar{\lambda}_L/2}\Gamma_L(1 - f_L), \quad \Sigma_L^{++} = i\Gamma_L(f_L - \frac{1}{2}), \\ \Sigma_R^{--} &= i\Gamma_R(f_R - \frac{1}{2}), \quad \Sigma_R^{-+} = ie^{i\bar{\lambda}_R/2}\Gamma_R f_R, \quad \Sigma_R^{+-} = -ie^{-i\bar{\lambda}_R/2}\Gamma_R(1 - f_R), \quad \Sigma_R^{++} = i\Gamma_R(f_R - \frac{1}{2}). \end{aligned} \quad (\text{B5})$$

Then the solution of the Dyson equation is found as $\hat{G} = (\hat{g}^{-1} - \hat{\Sigma})^{-1}$. Let Δ equal to the determinant of $(\hat{g}^{-1} - \hat{\Sigma})$, which requires being solved first before the solution of the Dyson equation. To have the expression of \hat{G} brevity, we define some auxiliary quantities: $\tilde{\epsilon}_1^- = E - \epsilon_{1r}^-$, $\tilde{\epsilon}_1^+ = -(E - \epsilon_{1r}^+)$, $\tilde{\epsilon}_2^- = E - \epsilon_{2r}^-$, and $\tilde{\epsilon}_2^+ = -(E - \epsilon_{2r}^+)$. Now, let's turn to the expression of Δ ($\bar{\lambda} = \bar{\lambda}_L - \bar{\lambda}_R$):

$$\begin{aligned} \Delta &= [(\tilde{\epsilon}_1^- - \Sigma_L^{--})(\tilde{\epsilon}_1^+ - \Sigma_L^{++}) - \Sigma_L^{-+}\Sigma_L^{+-}] [(\tilde{\epsilon}_2^- - \Sigma_R^{--})(\tilde{\epsilon}_2^+ - \Sigma_R^{++}) - \Sigma_R^{-+}\Sigma_R^{+-}] + (\Sigma_L^{-+}\Sigma_R^{+-} + \Omega_{12}^-\Omega_{21}^+)(\Sigma_L^{+-}\Sigma_R^{-+} \\ &\quad + \Omega_{21}^-\Omega_{12}^+) - \Sigma_L^{-+}\Sigma_L^{+-}\Sigma_R^{-+}\Sigma_R^{+-} - \Omega_{21}^+\Omega_{12}^+(\tilde{\epsilon}_1^- - \Sigma_L^{--})(\tilde{\epsilon}_2^- - \Sigma_R^{--}) - \Omega_{12}^-\Omega_{21}^-(\tilde{\epsilon}_1^+ - \Sigma_L^{++})(\tilde{\epsilon}_2^+ - \Sigma_R^{++}). \end{aligned} \quad (\text{B6})$$

We divide Δ into two separate parts, i.e $\Delta = \Delta_i + \Delta_e$. The term Δ_e explicitly contains the counting field parameter λ_η and vanishes when setting $\lambda_\eta = 0$:

$$\Delta_e = \Gamma_L \Gamma_R [(\Omega - U\rho_{21}^-)(\Omega - U\rho_{12}^+)f_R(1 - f_L)(e^{-i\bar{\lambda}/2} - 1) + (\Omega - U\rho_{21}^+)(\Omega - U\rho_{12}^-)f_L(1 - f_R)(e^{i\bar{\lambda}/2} - 1)]. \quad (\text{B7})$$

And the term Δ_i , which implicitly contains the parameter λ and is involved in the expression of $T_\lambda(E)$, gives the following formidable expression

$$\begin{aligned} \Delta_i &= \left\{ \left[E - \epsilon_1 - U\rho_{22}^- - i\Gamma_L(f_L - \frac{1}{2}) \right] \left[E - \epsilon_1 - U\rho_{22}^+ + i\Gamma_L(f_L - \frac{1}{2}) \right] + \Gamma_L^2 f_L(1 - f_L) \right\} \\ &\quad \times \left\{ \left[E - \epsilon_2 - U\rho_{11}^- - i\Gamma_R(f_R - \frac{1}{2}) \right] \left[E - \epsilon_2 - U\rho_{11}^+ + i\Gamma_R(f_R - \frac{1}{2}) \right] + \Gamma_R^2 f_R(1 - f_R) \right\} \\ &\quad - (\Omega - U\rho_{12}^+)(\Omega - U\rho_{21}^+) \left[E - \epsilon_1 - U\rho_{22}^- - i\Gamma_L(f_L - \frac{1}{2}) \right] \left[E - \epsilon_2 - U\rho_{11}^- - i\Gamma_R(f_R - \frac{1}{2}) \right] \\ &\quad - (\Omega - U\rho_{12}^-)(\Omega - U\rho_{21}^-) \left[E - \epsilon_1 - U\rho_{22}^+ + i\Gamma_L(f_L - \frac{1}{2}) \right] \left[E - \epsilon_2 - U\rho_{11}^+ + i\Gamma_R(f_R - \frac{1}{2}) \right] \\ &\quad + \Gamma_L \Gamma_R [(\Omega - U\rho_{21}^-)(\Omega - U\rho_{12}^+)f_R(1 - f_L) + (\Omega - U\rho_{21}^+)(\Omega - U\rho_{12}^-)f_L(1 - f_R)] \\ &\quad + (\Omega - U\rho_{12}^-)(\Omega - U\rho_{12}^+)(\Omega - U\rho_{21}^-)(\Omega - U\rho_{21}^+). \end{aligned} \quad (\text{B8})$$

This complicated expression illustrates the complexity of coupled dots system in theory. However, Δ_i can be reduced to a fairly simple result, when setting $\lambda = 0$, which is the naturally requirement in deriving the current and shot noise formulas, as

$$\Delta^{(0)} \equiv \Delta_i(\lambda = 0) = \left| (E - \epsilon_1 - U\rho_{22}^{(0)} + \frac{i}{2}\Gamma_L)(E - \epsilon_2 - U\rho_{11}^{(0)} + \frac{i}{2}\Gamma_R) - (\Omega - U\rho_{12}^{(0)})(\Omega - U\rho_{12}^{(0)}) \right|^2, \quad (\text{B9})$$

where $|\dots|$ stands for the plural mode. Now, it is natural to give the expression of \hat{G} from the Dyson equation

$$\hat{G} = \frac{1}{\Delta} \begin{pmatrix} \tilde{\epsilon}_1^- - \Sigma_L^{--} & \Sigma_L^{+-} & -\Omega_{12}^- & 0 \\ \Sigma_L^{+-} & \tilde{\epsilon}_1^+ - \Sigma_L^{++} & 0 & \Omega_{12}^+ \\ -\Omega_{21}^- & 0 & \tilde{\epsilon}_2^- - \Sigma_R^{--} & \Sigma_R^{+-} \\ 0 & \Omega_{21}^+ & \Sigma_R^{+-} & \tilde{\epsilon}_2^+ - \Sigma_R^{++} \end{pmatrix}^{-1} \quad (\text{B10})$$

By the way, in the derivation for the adiabatic potential, G_{11}^{+-} and G_{11}^{+} are needed. So we give their explicit expressions here as

$$G_{11}^{+-} = -\frac{1}{\Delta} [\Sigma_L^{+-} (\tilde{\epsilon}_2^- - \Sigma_R^{--}) (\tilde{\epsilon}_2^+ - \Sigma_R^{++}) - \Omega_{21}^- \Omega_{12}^+ \Sigma_R^{+-} - \Sigma_L^{+-} \Sigma_R^{+-} \Sigma_R^{+-}], \quad (\text{B11})$$

$$G_{11}^{+} = -\frac{1}{\Delta} [\Sigma_L^{+-} (\tilde{\epsilon}_2^- - \Sigma_R^{--}) (\tilde{\epsilon}_2^+ - \Sigma_R^{++}) - \Omega_{21}^+ \Omega_{12}^- \Sigma_R^{+-} - \Sigma_L^{+-} \Sigma_R^{+-} \Sigma_R^{+-}]. \quad (\text{B12})$$

As illustrated before, the solution of \hat{G} depends on the self-consistent parameters $\rho_{\alpha\beta}$, which have to be calculated by the self-consistent equations. We find that doing the expansion with respect to the parameter λ_L and λ_R for those well-defined matrix in the Dyson equation makes the solution feasible in practice. Therefore, we elaborate the expansion calculations here, especially specifying Eqs.(52) – (54):

$$\begin{aligned} \hat{g}^{-1} = & \begin{pmatrix} E - \epsilon_1 - U\rho_{22}^{(0)} & 0 & -\Omega + U\rho_{21}^{(0)} & 0 \\ 0 & -(E - \epsilon_1 - U\rho_{22}^{(0)}) & 0 & \Omega - U\rho_{21}^{(0)} \\ -\Omega + U\rho_{12}^{(0)} & 0 & E - \epsilon_2 - U\rho_{11}^{(0)} & 0 \\ 0 & \Omega - U\rho_{12}^{(0)} & 0 & -(E - \epsilon_2 - U\rho_{11}^{(0)}) \end{pmatrix} \\ & + \sum_{\eta=L,R} \left(\frac{i}{2} \lambda_\eta \right) \begin{pmatrix} -U\rho_{22,\eta}^{-(1)} & 0 & U\rho_{21,\eta}^{-(1)} & 0 \\ 0 & U\rho_{22,\eta}^{+(1)} & 0 & -U\rho_{21,\eta}^{+(1)} \\ U\rho_{12,\eta}^{-(1)} & 0 & -U\rho_{11,\eta}^{-(1)} & 0 \\ 0 & -U\rho_{12,\eta}^{+(1)} & 0 & U\rho_{11,\eta}^{+(1)} \end{pmatrix} + o(\lambda_L, \lambda_R), \end{aligned} \quad (\text{B13})$$

or being written in an abbreviating symbolic way as $\hat{g}^{-1} = \hat{g}^{-1(0)} + \sum_\eta \hat{g}_\eta^{-1(1)} \left(\frac{i}{2} \lambda_\eta \right) + o(\lambda_L, \lambda_R)$,

$$\begin{aligned} \hat{\Sigma} = & \begin{pmatrix} i\Gamma_L(f_L - \frac{1}{2}) & -i\Gamma_L f_L & 0 & 0 \\ i\Gamma_L(1 - f_L) & i\Gamma_L(f_L - \frac{1}{2}) & 0 & 0 \\ 0 & 0 & i\Gamma_R(f_R - \frac{1}{2}) & -i\Gamma_R f_R \\ 0 & 0 & i\Gamma_R(1 - f_R) & i\Gamma_R(f_R - \frac{1}{2}) \end{pmatrix} + \left(\frac{i}{2} \lambda_L \right) \begin{pmatrix} 0 & -i\Gamma_L f_L & 0 & 0 \\ -i\Gamma_L(1 - f_L) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \\ & + \left(\frac{i}{2} \lambda_R \right) \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -i\Gamma_R f_R & 0 \\ 0 & -i\Gamma_R(1 - f_R) & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + o(\lambda_L, \lambda_R), \end{aligned} \quad (\text{B14})$$

or symbolized as $\hat{\Sigma} = \hat{\Sigma}^{(0)} + \sum_\eta \hat{\Sigma}_\eta^{(1)} \left(\frac{i}{2} \lambda_\eta \right) + o(\lambda_L, \lambda_R)$, and

$$\hat{G} = \begin{pmatrix} G_{11}^{(0)} & G_{11}^{(0)} & G_{12}^{(0)} & G_{12}^{(0)} \\ G_{11}^{(0)} & G_{11}^{(0)} & G_{12}^{(0)} & G_{12}^{(0)} \\ G_{21}^{(0)} & G_{21}^{(0)} & G_{22}^{(0)} & G_{22}^{(0)} \\ G_{21}^{(0)} & G_{21}^{(0)} & G_{22}^{(0)} & G_{22}^{(0)} \end{pmatrix} + \sum_{\eta=L,R} \left(\frac{i}{2} \lambda_\eta \right) \begin{pmatrix} G_{11,\eta}^{--(1)} & G_{11,\eta}^{+- (1)} & G_{12,\eta}^{--(1)} & G_{12,\eta}^{+- (1)} \\ G_{11,\eta}^{+- (1)} & G_{11,\eta}^{++ (1)} & G_{12,\eta}^{+- (1)} & G_{12,\eta}^{++ (1)} \\ G_{21,\eta}^{--(1)} & G_{21,\eta}^{+- (1)} & G_{22,\eta}^{--(1)} & G_{22,\eta}^{+- (1)} \\ G_{21,\eta}^{+- (1)} & G_{21,\eta}^{++ (1)} & G_{22,\eta}^{+- (1)} & G_{22,\eta}^{++ (1)} \end{pmatrix} + o(\lambda_L, \lambda_R), \quad (\text{B15})$$

or symbolized as $\hat{G} = \hat{G}^{(0)} + \sum_\eta \hat{G}_\eta^{(1)} \left(\frac{i}{2} \lambda_\eta \right) + o(\lambda_L, \lambda_R)$. Inserting all the expansions above in the Dyson equation (43), one obtains

$$\left[\hat{g}^{-1(0)} + \sum_\eta \hat{g}_\eta^{-1(1)} \left(\frac{i}{2} \lambda_\eta \right) \right] \left[\hat{G}^{(0)} + \sum_\eta \hat{G}_\eta^{(1)} \left(\frac{i}{2} \lambda_\eta \right) \right] = \hat{I} + \left[\hat{\Sigma}^{(0)} + \sum_\eta \hat{\Sigma}_\eta^{(1)} \left(\frac{i}{2} \lambda_\eta \right) \right] \left[\hat{G}^{(0)} + \sum_\eta \hat{G}_\eta^{(1)} \left(\frac{i}{2} \lambda_\eta \right) \right]. \quad (\text{B16})$$

Then if the second and higher orders are ignored, Eq.(B16) gives the zero-order and first-order Dyson equations, i.e. Eqs.(55) - (57).

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